

Electric Fields, Potentials, and Forces

Numbers in italics refer to the Exercise number in the 12th Edition of Young & Freedman's "University Physics". Answers to odd-numbered questions are at the back of the book. Do refresh your knowledge of vectors.

- [21.7] Two small plastic spheres are given positive electrical charges. When they are 15.0 cm apart, the repulsive force between them has magnitude 0.220 N. What is the charge on each sphere (a) if the two charges are equal? (b) if one sphere has four times the charge of the other?
- [21.28] An electron is released from rest in a uniform electric field. The electron accelerates vertically upward, traveling 4.50 m in the first 3.00 μs after it is released. (a) What are the magnitude and direction of the electric field? (b) Are we justified in ignoring the effects of gravity? Justify your answer quantitatively.
- [21.79] Three identical point charges q are placed at each of three corners of a square of side L . Find the magnitude and direction of the net force on a point charge $-3q$ placed (b) at the vacant corner of the square. Draw a free-body diagram showing the forces exerted on the $-3q$ charge by each of the other three charges, together with the resultant net force. [You do not need to do part (a) of this question, which is to find the net force if the charge $-3q$ is placed at the centre of the square, but you are free to do this if you wish!]
- [21.96] Positive charge Q is uniformly distributed around a semicircular ring of radius a that lies in the upper half of the $x - y$ plane with its centre of curvature at the origin. [i.e., half of a circular ring of charge.] Find the electric field (magnitude and direction) at the origin. [Hint: Divide the ring into pieces of elemental length $a d\theta$ and charge dQ . Find the charge per unit length around the semicircle. Then use symmetry and superposition to find \mathbf{E} . Check that your answer makes sense.]
- In lecture we saw that an electric dipole lying along the x -axis with a charge $+q$ at $x = -d/2$ and a charge $-q$ at $x = d/2$ has a field at a point $x = X$, $y = 0$ with $X < -d/2$ given by

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0} \left[\frac{-1}{\left(X + \frac{d}{2}\right)^2} + \frac{1}{\left(X - \frac{d}{2}\right)^2} \right] \hat{\mathbf{x}}$$

Consider points X which are a long way from the dipole, so that the denominators can be written in terms of $X \left(1 \pm \frac{d}{2X}\right)$ with $|d/2X| \ll 1$. Use these to expand the expressions in \mathbf{E} using the binomial theorem to show that

$$E_x = \frac{qd}{2\pi\epsilon_0 X^3}$$

The numerator is the dipole moment $p \equiv qd$. Does this make sense in terms of direction of \mathbf{E} and dependence on X ? [Hint: Remember that $X < 0$. Why doesn't the field fall off as $1/X^2$?]

Remember ISEE: *Identify* the relevant concepts, including the target variable (what you need to find); *Set Up* the problem, usually with the help of a sketch; *Execute* the calculation needed to find the value of the target variable; finally *Evaluate* your answer: does it correspond to the symmetry of the problem, does the answer seem reasonable?

Some possibly useful information and formulae are on the next page.

$$\mathbf{F}_{1on2} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\mathbf{r}_2 - \mathbf{r}_1|^2} \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|}$$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{|\mathbf{r} - \mathbf{r}_q|^2} \frac{\mathbf{r} - \mathbf{r}_q}{|\mathbf{r} - \mathbf{r}_q|}$$

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$$

Principle of Superposition: \mathbf{F} or \mathbf{E} due to collection of charges is equal to the sum over all the charges of the \mathbf{F} or \mathbf{E} due to each individual charge.

Symbol	Value	Units
ϵ_0	8.85×10^{-12}	$\text{C}^2/\text{N m}^2$
$1/4\pi\epsilon_0$	8.99×10^9	$\text{N m}^2/\text{C}^2$
e	1.60×10^{-19}	C
m_e	9.11×10^{-31}	kg
g	9.81	m/s^2