UNIVERSITY OF LONDON

BSc/MSci EXAMINATION June 2004

for Internal Students of Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant Examination for the Associateship

ELECTRICITY & MAGNETISM

For First-Year Physics Students

Monday 7th June 2004: 14.00 to 16.00

Answer ALL parts of Section A and TWO questions from Section B.

Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Write your CANDIDATE NUMBER clearly on each of the THREE answer books provided.

If an electronic calculator is used, write its serial number in the box at the top right hand corner of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the horizontal box on the front cover of its corresponding answer book.

Hand in THREE answer books even if they have not all been used.

You are reminded that the Examiners attach great importance to legibility, accuracy and clarity of expression.

Turn over for questions

SECTION A

1. (i) An electron is in a circular orbit of radius a around a proton. Find the ratio of potential to kinetic energy. If a is the Bohr radius (5.3×10^{-11} m), find the velocity. Ignore any radiation of energy associated with the electron's acceleration.

[7 marks]

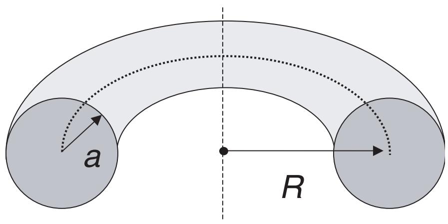
(ii) The plates of a parallel plate capacitor are square (50 mm × 50 mm), and are separated by 0.2 mm. Calculate the capacitance.

An insulating sheet of dielectric constant 3.5, thickness 0.1 mm, and the same size and shape as the plates is placed between and parallel to the plates. How is the capacitance changed?

Neglect edge effects.

[8 marks]

(iii) Use Ampère's Law to deduce the magnetic field $\boldsymbol{B}$ at points within a toroidal solenoid of major radius R and minor radius $a (< R)$ (see diagram), as a function of the current I and the number of turns N.



Show that the difference between the maximum and minimum magnitude of $\boldsymbol{B}$ within the torus is

$$\Delta B = \frac{\mu_0 N I a}{\pi (R^2 - a^2)}$$

Deduce the magnitude of $\boldsymbol{B}$ inside a long *straight* solenoid with 120 turns per cm and carrying a current of 50 mA.

[7 marks]

(iv) A circular loop of wire of area $2 \times 10^{-4}\,\mathrm{m}^2$ lies with its plane normal to a uniform magnetic field of 0.4 T. Find the magnetic flux through the loop.

The loop is rotated by 180° at a uniform angular velocity of 5 rad/s, about a line perpendicular to the magnetic field. If the resistance of the loop is 2 kΩ, how does the current vary in time, and what is the total charge that flows in the loop?

[7 marks]

(v) Short pulse, high intensity lasers can be used to strip many electrons from a molecule in a very short space of time. This leaves positively charged ions in close proximity to each other, and repulsion between these ions then results in an energetic "Coulomb Explosion". A hydrogen iodide (HI) molecule with a bond length $r_o = 1.1 \times 10^{-10}$ m is laser ionised and 5 electrons are removed from the iodine and 1 electron from the hydrogen.

(a) Write down a simple expression for the electric field due to the I^{5+} ion alone as a function of radial distance r from the ion.

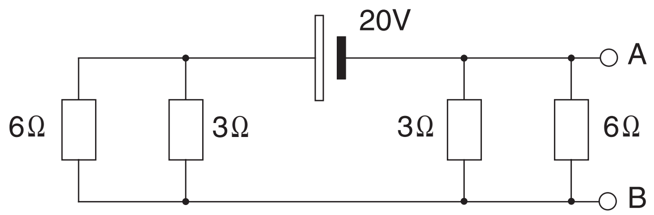
(b) Assume that the heavy I^{5+} ion is fixed and the much lighter H^{+} ion is mobile. What is the final kinetic energy of the H^{+} ion in eV after the Coulomb Explosion?

You may use $1/4\pi\varepsilon_0 = 9 \times 10^9$ m/F.

[7 marks]

(vi) (a) State Kirchhoff's voltage law as concisely as possible.

(b) Find a Thévenin equivalent circuit for the points A–B in the network shown below.



[7 marks]

(vii) A voltage $V(t) = 50\cos(\omega t + \pi/6)$ is applied across a circuit element and results in a current flow given by $I(t) = 2\cos(\omega t - \pi/3)$.

(a) Find a complex exponential expression for the impedance $\underline{\mathbf{Z}}$ of the circuit element.

(b) Is the circuit element a pure resistor, inductor, capacitor or none of these three? Give a brief reason for your answer.

[7 marks]

[TOTAL 50 marks]

SECTION B

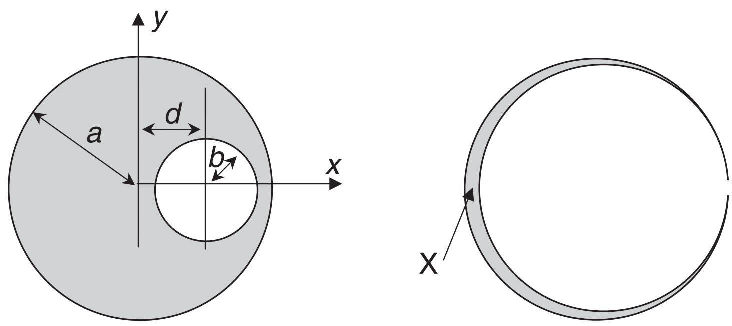
2. (i) Explain what is meant by the *Principle of Superposition*. [2 marks]

(ii) A solid insulating sphere of radius a carries a uniform charge per unit volume of ρ. Use Gauss's Flux Law to obtain expressions for the electric field inside and outside the sphere. Hence show that the electric potential (defined with the zero of potential at $r = \infty$) is given by

$$V(r) = \begin{cases} \dfrac{\rho a^3}{3\varepsilon_0 r}, & r > a \\ \dfrac{\rho a^2}{6\varepsilon_0}\left(3 - \dfrac{r^2}{a^2}\right), & r < a. \end{cases}$$

[8 marks]

(iii) (a) A spherical section of radius b is removed from the sphere to leave a hollow cavity as shown in diagram (A) below. The centre of the original sphere lies at the coordinate origin, and the cavity centre at $(x, y) = (d, 0)$.



(A) (B)

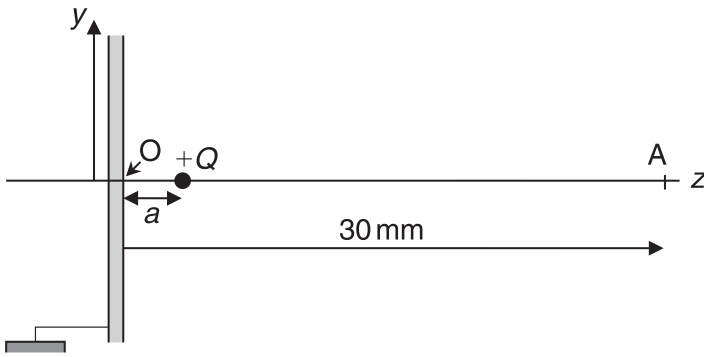
By using the Principle of Superposition at a general point (x, y) within the cavity, show that the magnitude of the electric field is $\boldsymbol{E} = (\rho d/3\varepsilon_0)\boldsymbol{i}$ throughout the cavity, where $\boldsymbol{i}$ is the unit vector in the x-direction. [7 marks]

(b) In diagram B, the hollow cavity just touches the surface of the larger sphere (i.e. $a = d + b$). Calculate the field in the cavity if $\rho = 2\,\mu\text{C/m}^3$ and the thickness of the wall at X is 2 mm. [3 marks]

[TOTAL 20 marks]

3. (i) Explain what is meant by the *Method of Images*. [2 marks]

(ii) Consider a charge $+Q$ situated on the z-axis at a distance a from a grounded conducting plate of infinite extent in the x–y plane (see diagram). The coordinate origin O is therefore at the closest point on the plate to the $+Q$ charge.



Show that the charge density on the surface of the plate as a function of distance (y) from O is

$$\sigma(y) = \frac{Qa}{2\pi(a^2 + y^2)^{3/2}}$$

[8 marks]

(iii) If $Q = 100\,\text{pC}$ and $a = 3\,\text{mm}$, calculate:

(a) the field adjacent to the surface of the plate at O;

(b) the charge density on the surface of the plate at $y = 2\,\text{mm}$;

(c) the magnitude and direction of the electric field at A, which lies on the z–axis at 30 mm from O (see diagram).

[9 marks]

(iv) What is the field in the region to the left of the plate $(z < 0)$? [1 mark]

[TOTAL 20 marks]

4. The Biot-Savart law gives the contribution to the magnetic field at point P from a current element $I\boldsymbol{ds}$ in the form

$$d\boldsymbol{B} = \frac{\mu_0}{4\pi}\frac{I\boldsymbol{ds}\times\hat{\boldsymbol{r}}}{r^2}$$

where r is the distance from the current element to P, and $\hat{\boldsymbol{r}}$ the unit vector in this direction.

(i) Current I flows in a circular conducting loop of radius a lying in the x–y plane with its centre at the origin. Show that the magnetic field on the axis of the loop at distance z from the origin is

$$\boldsymbol{B} = \frac{\mu_0 I a^2 \boldsymbol{k}}{2(z^2+a^2)^{3/2}}$$

where $\boldsymbol{k}$ is the unit vector in the z-direction. Which sense of current circulation in the loop corresponds to positive I? [7 marks]

(ii) The loop is replaced by a thin insulating *disk* of thickness t and the same radius $a(\gg t)$ as the loop, which rotates at angular velocity ω about the z-axis. If the disk carries uniform charge density ρ, show that the magnetic field at the centre of the disk ($x = y = z = 0$) is

$$\boldsymbol{B} = \frac{\mu_0 \rho t \omega a \boldsymbol{k}}{2}.$$

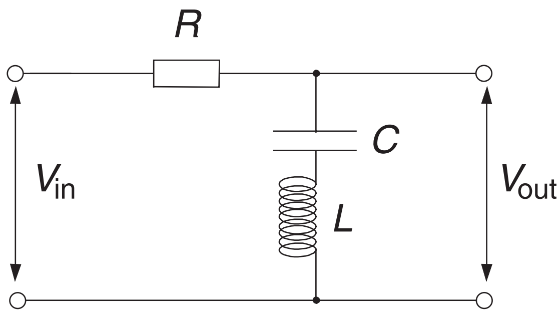
[7 marks]

(iii) Calculate the magnetic field at the centre of a conducting loop of radius 50 mm resulting from a current of 2 mA. [3 marks]

(iv) If a uniformly-charged disk of the same radius rotates at 25 revolutions per second, what charge must it carry to produce the same magnetic field at its centre. [3 marks]

[TOTAL 20 marks]

5. (i) A "notch" filter can be constructed as shown below. Derive a simple expression for $\underline{\mathbf{Z}}_{\text{in}}$, the input impedance seen by a signal connected to the V_{in} terminals and express this as separate magnitude and phase angle terms. [5 marks]



(ii) Find a simple expression for $\underline{\mathbf{V}}_{\text{out}}$ as a function of L, C, R and $\underline{\mathbf{V}}_{\text{in}}$. Use this to determine the resonant frequency of the filter and the magnitude of $V_{\text{out}}(t)$ in the high and low frequency limits. [10 marks]

(iii) A notch filter of the type shown above is required to remove 20 kHz "noise" from a signal. If $C = 0.1\,\mu$F what value of inductor is required ? [5 marks]

(iv) Derive (but do not solve) a quadratic equation which gives the "half frequencies" $\omega_{\pm 1/2}$ of the filter, where the magnitude of V_{out} is half that of V_{in}. [5 marks]

[TOTAL 25 marks]

End