First-Year Mathematics

Classwork 5 | Line Integrals | February 4, 2005

1. Evaluate the integrals

$$I_1 = \int (xy + y)\, dx \qquad \text{and} \qquad I_2 = \int (xy + y)\, dy$$

from $(0, 0)$ to $(2, 1)$ along the following paths:

(a) $y = \frac{1}{2}x$,

(b) $x = 0$ (from $y = 0$ to $y = 1$) and then $y = 1$ (from $x = 0$ to $x = 2$),

(c) $x = 2t$, $y = t^2$, for $0 \leq t \leq 1$.

Ans: (a) $I_1 = \frac{7}{3}$, $I_2 = \frac{7}{6}$; (b) $I_1 = 4$, $I_2 = \frac{1}{2}$; (c) $I_1 = \frac{5}{3}$, $I_2 = \frac{13}{10}$.

2. Evaluate the integral

$$I = \int (x^2 + y)\, dx$$

from $(0, 0)$ to $(1, 1)$ along the path $y = x^2$. Convert this to an integral over y and show that you get the same answer.

Ans: $I = \frac{2}{3}$.

3. Evaluate the integral

$$\oint y\, dx$$

along a path consisting of straight line segments connecting the points $(1, 1)$, $(1, 2)$, $(2, 2)$, and $(2, 1)$ in a clockwise direction to form a square. Compare your answer with the area enclosed by these lines.

4. Consider the closed curve parametrized by

$$x = a\cos t, \qquad y = -b\sin t\,,$$

where $0 \leq t < 2\pi$. Identify this curve if $b = a$ and if $b \neq a$. Evaluate the integral

$$\oint y\, dx$$

around this curve and provide an interpretation of your result.