ANSWERS to Lecture 16 problems

1. (i) $\begin{pmatrix} 12 & 3 \\ 9 & 6 \end{pmatrix}$ (ii) 5 (iii) 45.

   A determinant is multiplied by *f* if all members of one row are multiplied by *f*. Therefore, if all members of <u>all *n*</u> rows are multiplied by *f*, which is what happens if the parent matrix is multiplied by *f*, the determinant will be multiplied by f^n.

2. (i) The eigenvalues are 6 and 1; the respective eigenvectors are $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$. The eigenvectors are orthogonal because the parent matrix is symmetric. Divide both eigenvectors by √3 to normalise them.

   (ii) The eigenvalues are −2 and 4; the respective eigenvectors are $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 7 \\ 1 \end{pmatrix}$. Divide the eigenvectors by √2 and √50 respectively to normalise. Note that the eigenvectors are not orthogonal in this case.

3. $\mathbf{A}^2\mathbf{r} = \mathbf{A}(\mathbf{Ar}) = \mathbf{A}(\lambda\mathbf{r}) = \lambda\mathbf{Ar} = \lambda^2\mathbf{r}$. The eigenvalues of $\mathbf{A}^2$ are therefore the square of the eigenvalues of **A**, and so are 4 and 16 in this case.

4. (i) The matrix is already diagonal, so the eigenvalues and eigenvectors are

   3 and $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, 5 and $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, 27 and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

   (ii) The eigenvalues and eigenvectors are 2 and $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, 1 and $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, −1 and $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$.

   (iii) The eigenvalues and eigenvectors are 3 and $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$, 2 and $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, −2 and $\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$.

5. (i) 2√2 and 45° (ii) √10 and 108.43° (iii) 2√2 and −45°

   (iv) 4√5 and 153.43° (v) √5/2 and 63.43°. (vi) 2/√5 and −153.43°.

6. (i) 32768*i* (ii) 0.0028 – 0.0096*i* (iii) −32768*i*.

7. (i) $\cosh x + \sinh x = \dfrac{e^x + e^{-x}}{2} + \dfrac{e^x - e^{-x}}{2} = e^x$

(ii) $\cosh x - \sinh x = \dfrac{e^x + e^{-x}}{2} - \dfrac{e^x - e^{-x}}{2} = e^{-x}$

(iii) Multiply parts (i) and (ii).

(iv) $\sin iy = \dfrac{e^{i(iy)} - e^{-i(iy)}}{2i} = -i\dfrac{e^{-y} - e^{y}}{2} = i\sinh y$.

(v)

$$\sin(x+iy) = \frac{e^{-y+ix} - e^{y-ix}}{2i} = \frac{e^{-y}(\cos x + i\sin x) - e^{y}(\cos x - i\sin x)}{2i}$$

$$= i\cos x\frac{e^{y} - e^{-y}}{2} + \sin x\frac{e^{y} + e^{-y}}{2} = \sin x\cosh y + i\cos x\sinh y.$$

8. (i) $\log_e(2^{i/2}) = \frac{1}{2}i\log_e 2$, so $2^{i/2} = e^{i(\log_e 2)/2} = e^{0.347i} = 0.941 + 0.340i$,

or set $2 = e^{\log_e 2}$ at the outset.

(ii) Since $i = e^{i\pi/2}$, $\log_e i = i\pi/2$.

(iii)

$$(2^{1/2}e^{i\pi/4})^{1+i} = 2^{(1+i)/2}e^{i(1+i)\pi/4} = 2^{1/2}2^{i/2}e^{i\pi/4}e^{-\pi/4}$$

$$= 2^{1/2}e^{-\pi/4}e^{i(\pi/4+0.347)} = 0.274 + 0.584i$$