Lines, Planes, Rotations

1. A plane is defined by the equation $5x-4y-3z=10$. Find (a) the unit normal vector, (b) the shortest distance from the origin to the plane, (c) the shortest distance from the point (1, 3, 5) to the plane.

2. Consider the two planes $5x-4y-3z=10$ and $-2x+y+z=2$. Find a normal to the second plane (you found a normal to the first in question 1. Hence find an equation for the line of intersection of the two planes in both vector and Cartesian form.

3. What can you say about the intersection of the plane $x-2y-z=14$ with the two in question 2?

4. Find the shortest distance from (1, –2, 0) to the line joining (–2, 1, 2) to (5, 5, 5).

5. For what value of α do the two lines $\mathbf{r}=(\mathbf{i}+2\mathbf{j}+3\mathbf{k})+\lambda(3\mathbf{i}+2\mathbf{j}+\mathbf{k})$ and $\mathbf{r}=(\alpha\mathbf{i}+\mathbf{j}+\mathbf{k})+\mu(2\mathbf{i}-3\mathbf{j}-4\mathbf{k})$ intersect?

6. (a) Write down the matrix of the transformation defined by $X=2x+3y,\quad Y=x-y.$

   (b) Write down the equations for the transformation whose matrix is $\begin{pmatrix}7 & -4\\ 2 & 0\end{pmatrix}$.

7. Calculate the transformed position of the point (2, 1) under the following transformations:
   (a) Factor ×2 shrinkage in the *y*-direction,
   (b) Enlargement (all directions) of ×3,
   (c) Reflection in the *x*-axis.

8. (a) The 3×3 matrix for a rotation of θ about the z-axis is of the form $\begin{pmatrix}\cos\theta & \pm\sin\theta & 0\\ \mp\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{pmatrix}$. Which signs apply if positive θ corresponds to clockwise rotation about the +z axis i.e. clockwise looking in the +z direction?

   Find the analogous matrices for (b) a clockwise rotation about the +*x* axis and (c) a clockwise rotation about the +*y* axis.

9. Find the resulting vector if $\mathbf{r}=\begin{pmatrix}1\\2\\3\end{pmatrix}$ is rotated about the +*z*-axis by (a) 45° clockwise and (b) 45° anti-clockwise. Check that the operations preserve the magnitude of the vector.

10. The vector in the previous question is rotated first by 45° clockwise about the +*y*-axis and then by 45° anticlockwise about the +*x*-axis. Find the new vector. Use the same sense convention as in the previous questions. Check that the operation preserves the magnitude of the vector.

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* This sheet covers material presented in Lectures 13 and 14.