Singular Matrices and Linear Equations

1. Which of the following matrices are non-singular? For each that is, find the inverse:

   (a) $\begin{pmatrix} 0 & 2 \\ -2 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} 6 & -4 \\ -3 & 2 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}$

2. Which of the following matrices are singular? In those cases where a matrix can be seen by inspection to be singular, give your reasoning.

   (a) $\begin{pmatrix} 8 & 6 & 3 \\ 5 & 8 & 4 \\ 5 & 4 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 7 & 0 \\ 3 & -5 & 6 \\ 5 & 4 & 2 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 2 & 1 \\ -2 & 4 & -1 \\ -1 & 14 & 1 \end{pmatrix}$

   (d) $\begin{pmatrix} 4 & 4 & 4 \\ 2 & 1 & 2 \\ -1 & -1 & 1 \end{pmatrix}$ (e) $\begin{pmatrix} 3.5 & -7.2 & 2.1 & 4.4 \\ 5.3 & 6.2 & 0 & -6.2 \\ 3.5 & -7.2 & 2.1 & 4.4 \\ 1.7 & 0 & -5.3 & 0 \end{pmatrix}$ (f) $\begin{pmatrix} 3 & -5 & 0 & -1 \\ 2 & 1 & 7 & 4 \\ 0 & 6 & -4 & 2 \end{pmatrix}$

3. Consider the set of linear equations

   $$-x + 2y + 3z = k_1$$
   $$2x + y - 4z = k_2$$
   $$-x - 2y + z = k_3$$

   Write down the matrix of the coefficients and find in sequence (a) the determinant, (b) the matrix of the cofactors, (c) the adjoint, (d) the inverse, and (e) the general solution. Check your result for the inverse by making sure that, when you multiply it by the original matrix, you get a unit matrix.

4. Obtain the solution of the equations in the previous question for the following values of $\mathbf{k} = \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix}$: (a) $\mathbf{k} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ (b) $\mathbf{k} = \begin{pmatrix} 5 \\ -8 \\ 0 \end{pmatrix}$ (c) $\mathbf{k} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$:

5. (harder) Find the determinant of the matrix $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 5 & 7 & 5 & 5 \\ -1 & 3 & 2 & -1 \\ 3 & -2 & 5 & 4 \end{pmatrix}$. You will probably need to refer to Fact Sheet H.