Determinants

1. Determine which of the following determinants are zero. For those that are zero, try to identify what characteristic of the determinant ensures that it is zero.

(a) $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 6 & 12 \\ -5 & 10 & -15 \end{vmatrix}$ (b) $\begin{vmatrix} 0 & -1 & 0 \\ 0 & 5 & 3 \\ 2 & 0 & 0 \end{vmatrix}$ (c) $\begin{vmatrix} 7 & 3 & 2 \\ 6 & 1 & -1 \\ 1 & 2 & 3 \end{vmatrix}$ (d) $\begin{vmatrix} 0 & 7 & 0 \\ 3 & -5 & 6 \\ 2 & 3 & -4 \end{vmatrix}$

2. In the lecture, the 3×3 determinant $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ was expanded about the first column in the form $a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - c_2c_1)$. Write down this result in a form expanded about the first row instead.

3. Written out in full, the 3×3 determinant of question 2 reads

$$a_1b_2c_3 - a_1b_3c_2 - a_2b_1c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c_1.$$

(a) How many terms are there in the sum?

(b) How many multiplications have to be carried out to find each terms?

(c) How many multiplications have to be carried out to calculate the entire determinant?

(d) Answer (a)-(c) for a 4×4 determinant

(e) Answer (a)-(c) for an $n \times n$ determinant.

(f) How many multiplications would be required to evaluate a 20×20 determinant?

(g) A modest PC can carry out 10^8 multiplications per second. How long would it take it to evaluate a 20×20 determinant in this way?

4. If you want a challenge, try to show that the following determinants are zero. But don't be too depressed if you fail! Many questions in mathematics are simple (even "trivial") once you see the point, but far from trivial until you do!

(a) $\begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix}$ (b) $\begin{vmatrix} 1 & a & a^2 & a^3 + bcd \\ 1 & b & b^2 & b^3 + cda \\ 1 & c & c^2 & c^3 + dab \\ 1 & d & d^2 & d^3 + abc \end{vmatrix}$