ANSWERS for Lecture 7 problems

1. (a) $\begin{vmatrix} 4 & 2 \\ 1 & 5 \end{vmatrix} = 18$. (b) $\begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} = +18$; rows and columns have been exchanged.

   (c) $\begin{vmatrix} 1 & 5 \\ 4 & 2 \end{vmatrix} = -18$; the rows have been reversed, leading to a sign change.

   (d) $\begin{vmatrix} 8 & 2 \\ 2 & 5 \end{vmatrix} = 36$; the top row of the determinant in (a) has been doubled.

   (e) $\begin{vmatrix} 4 & 2 \\ 4 & 2 \end{vmatrix} = 0$; two identical rows.

   (f) $\begin{vmatrix} 4 & 2 \\ 5 & 7 \end{vmatrix} = 18$; row 1 of (a) has been added to row 2.

2. (a) $\begin{matrix} 3x+5y=14 \\ 2x+4y=10 \end{matrix}$; $x = 3$ and $y = 1$

   (b) $\begin{matrix} 3x-5y=8 \\ 7x+2y=12 \end{matrix}$; $x = \frac{76}{41}$ and $y = -\frac{20}{41}$

   (c) no solution; the equations represent the same line.

   (d) no solution; the lines are parallel.

3. (a) no solution; the determinant of the coefficients is zero.

   (b) the solution is a line. (c) yes; the determinant of the coefficients = 32.

4. (a) 50.

   (b) 10.

   (c) 240 = the product of the diagonal elements.

   (b) Working it out directly shows that the determinant is zero. A clever way to deduce this result is to note that, if you multiply each row in turn by –1 and then exchange rows and columns, the original determinant is recovered. But since, in general, this operation should reverse the sign, the value can only be zero!