Simultaneous Linear Equations & Determinants

1. Evaluate (a) $\begin{vmatrix} 4 & 2 \\ 1 & 5 \end{vmatrix}$ and (b) $\begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix}$.

   Note that the second is obtained from the first by exchanging rows and columns.

   Now evaluate each of the following 2×2 determinants, and decide how each is related to the determinants of parts (a) and (b):

   (c) $\begin{vmatrix} 1 & 5 \\ 4 & 2 \end{vmatrix}$ (d) $\begin{vmatrix} 8 & 2 \\ 2 & 5 \end{vmatrix}$ (e) $\begin{vmatrix} 4 & 2 \\ 4 & 2 \end{vmatrix}$ (f) $\begin{vmatrix} 4 & 2 \\ 5 & 7 \end{vmatrix}$

2. For each of the following pairs of equations, identify those that have a unique solution, and use Cramer's rule to find the solutions:

   (a) $\begin{matrix} 3x+5y=14 \\ 2x+4y=10 \end{matrix}$ (b) $\begin{matrix} 3x-5y=8 \\ 7x+2y=12 \end{matrix}$ (c) $\begin{matrix} 6x+3y=9 \\ 4x+2y=6 \end{matrix}$ (d) $\begin{matrix} 1.4x-1.2y=6.4 \\ -2.1x+1.8y=-4.7 \end{matrix}$

3. Determine whether the following sets of linear equations have a unique solution

   (a) $\begin{matrix} 8x+y+8z=12 \\ 6x+4y+4z=8 \\ 5x-y+6z=15 \end{matrix}$ (b) $\begin{matrix} 4x+7y-2z=10 \\ x-3y+2z=6 \end{matrix}$ (c) $\begin{matrix} x+y+z=1 \\ 2x-2y+2z=0 \\ 4x-4y-4z=-1 \end{matrix}$

4. Evaluate (a) $\begin{vmatrix} 4 & 1 & 2 \\ 7 & 2 & 0 \\ -2 & 3 & 0 \end{vmatrix}$ (b) $\begin{vmatrix} 3 & 2 & 4 \\ 5 & 4 & 8 \\ 8 & 2 & 9 \end{vmatrix}$

   (c) $\begin{vmatrix} 2 & 15 & -37 & 8 & 11 \\ 0 & 1 & 6 & 23 & -32 \\ 0 & 0 & 4 & 12 & -29 \\ 0 & 0 & 0 & 10 & 20 \\ 0 & 0 & 0 & 0 & 3 \end{vmatrix}$ (d) $\begin{vmatrix} 0 & 18.784 & -56.213 \\ -18.784 & 0 & 33.057 \\ 56.213 & -33.057 & 0 \end{vmatrix}$