ANSWERS to Lecture 5 problems (*Geometry 1*)

1. A possible equation for line A is $\mathbf{r} = (3\mathbf{i} + 4\mathbf{j}) + \lambda(\mathbf{r}_2 - \mathbf{r}_1) = (3 + 5\lambda)\mathbf{i} + (4 - 9\lambda)\mathbf{j}$, but note that the first vector on the rhs could be the position vector of *any* point on the line. This leads to

   $x = 3 + 5\lambda$

   $y = 4 - 9\lambda$

   and the result follows if λ is eliminated.

2. A possible equation for line B is $\mathbf{r} = \mu\mathbf{i} + (-2 + 3\mu)\mathbf{j}$ which corresponds to

   $x = \dfrac{y+2}{3}$

3. The direction ratios of line A are (5, –9) and the corresponding direction cosines are $(\dfrac{5}{\sqrt{106}}, \dfrac{-9}{\sqrt{106}})$.

   The direction ratios of line B are (1, 3) and the corresponding direction cosines are $(\dfrac{1}{\sqrt{10}}, \dfrac{3}{\sqrt{10}})$.

4. Reverse the direction cosines and reverse the sign of one of them. This leads to

   $\hat{\mathbf{n}}_1 = \dfrac{9}{\sqrt{106}}\mathbf{i} + \dfrac{5}{\sqrt{106}}\mathbf{j}$ and $\hat{\mathbf{n}}_2 = \dfrac{3}{\sqrt{10}}\mathbf{i} - \dfrac{1}{\sqrt{10}}\mathbf{j}$

5. The angle between the lines is the same as the angle between the normals. The result is $\cos\theta = \dfrac{22}{\sqrt{1060}} = 0.676$ yielding $\theta = 0.828 \text{ rad} = 47.5°$.

6. The perpendicular distance to line A is $p_1 = \hat{\mathbf{n}}_1.\mathbf{r}_1$ where $\mathbf{r}_1$ is any position vector on the line. The answer is $\dfrac{47}{\sqrt{106}}$. The corresponding result for line B is $\dfrac{2}{\sqrt{10}}$.

7. $\mathbf{r} = (2 + 5\lambda)\mathbf{i} + (1 - 3\lambda)\mathbf{j} + (-3 + 7\lambda)\mathbf{k}$. Eliminating λ between the component equations yields $\dfrac{x-2}{5} = -\dfrac{y-1}{3} = \dfrac{z+3}{7}$.