Complex Numbers 2 – ANSWERS (*corrected*)

2. (a) $(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}})$ (b) (-2.996, 6.547) (c) (0, –1)

3. (a) $\sqrt{41}$ and 51.34° (b) $\sqrt{53}$ and 105.95° (c) 1 and –90°

4. The modulus is 5 in all cases. The arguments are respectively

   0.442, 2.013, 3.584 (=–2.700), 5.154 (=–1.129), and 6.725 (= 0.442)

   which is where we started! Note that the numbers in brackets are obtained by subtracting 2π from the higher numbers. .

5. (a) $e^{i\pi/2}$ (b) $e^{-i\pi/2}$ (c) $\sqrt{2}\, e^{i\pi/4}$ (d) $2e^{-i\pi/3}$

6. Write the following complex numbers in the form $x + iy$

   (a) $-\frac{(1+i)}{\sqrt{2}}$ (b) $-\frac{(1+i)}{\sqrt{2}}$ (c) $(1.621 + 2.524i)$ (d) $\frac{1}{2\sqrt{3}} - \frac{i}{2}$

   (a) $e^{-i3\pi/4}$ (b) $e^{+i5\pi/4}$ (c) $3e^{i}$ (d) $\frac{2}{e^{i\pi/3}}$

7. Set $a = \cos\theta$ and $b = i\sin\theta$. The lhs is now $(\cos 3\theta + i\sin 3\theta)$. Equate real an imaginary parts to obtain

   $\cos 3\theta = \cos^3\theta + 3\cos\theta\sin^2\theta$ and $\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$

8. –665, –305, 55, 415, 775 & 1135; –1045, 35, 755 & 1115; –695, –335 & 25; –315 & 45. (Units are degrees)

   –43 & –15; –9, 5, 19, 33 & 61; –92, 6, 20, 48 & 62; 57 & 43; –39 & 143. (Units of π / 7 radians)