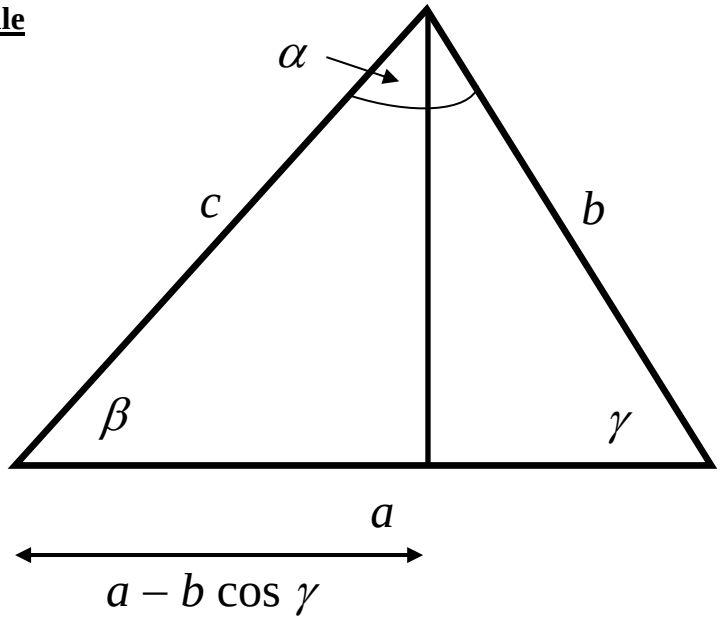
Fact Sheet D – cosine and sine rules

Simple proofs of these well-known rules are given below.

The Cosine Rule



Using Pythagoras on the left-hand right-angled triangle yields

$$c^2 = (a - b\cos\gamma)^2 + (b\sin\gamma)^2 = a^2 + b^2(\cos^2\gamma + \sin^2\gamma) - 2ab\cos\gamma$$

leading to

$$\boxed{c^2 = a^2 + b^2 - 2ab\cos\gamma}$$

The Sine Rule

From the diagram, it is evident that

$$c\sin\beta = b\sin\gamma$$

and hence

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

The result is readily extended to include the third side and the third angle to read

$$\boxed{\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}}$$