Fact Sheet A – Vector Basics

- A *vector* has magnitude and direction, whereas a *scalar* quantity has only magnitude.

- Vectors are written either in boldface type (e.g. **A**) or (esp. on the board) underlined (e.g. $\underline{A}$).

- The component of a vector along one of the coordinate axes (say x) is written A_x.

- The *magnitude* of a vector, associated with the length of the arrow in a diagrammatic representation, is written $|\mathbf{A}|$ or sometimes just A.

  Hence $|\mathbf{A}| \equiv A = \sqrt{A_x^2 + A_y^2}$ in 2D, or $|\mathbf{A}| \equiv A = \sqrt{A_x^2 + A_y^2 + A_z^2}$ in 3D.

- A vector of unit length (magnitude 1) is called a *unit vector*. The unit vector $\hat{\mathbf{a}}$ in the direction of a vector **A** is obtained by dividing the vector by its magnitude $|\mathbf{A}|$ so that

  $$\hat{\mathbf{a}} = \frac{\mathbf{A}}{|\mathbf{A}|}$$

  A circumflex is frequently used to denote a unit vector.

- The symbols **i**, **j**, and **k** are commonly used to represent unit vectors along the x, y, and z axes. Circumflexes are normally *not* applied for these special unit vectors.

- Using **i**, **j**, and **k**, a general vector **A** can be written

  $$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

- When a vector is used to define position relative to the origin, it is called a *position vector*. A point P with coordinates (x, y, z) is duly defined by the vector

  $$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

  Since $|\mathbf{r}| \equiv r = \sqrt{x^2 + y^2 + z^2}$, the unit vector in the direction of **r** is

  $$\hat{\mathbf{r}} = \frac{\mathbf{r}}{|\mathbf{r}|} \equiv \frac{\mathbf{r}}{r} = \frac{\mathbf{r}}{\sqrt{x^2 + y^2 + z^2}}$$

- Two vectors are identical if they have the same magnitude and direction. Line of action is not part of the definition of a vector.

- Multiplying a vector by a scalar factor is straightforward. For example, the vector 3**A** has three times the magnitude of **A** and points in the same direction. In terms of components, $3\mathbf{A} = (3A_x, 3A_y, 3A_z)$. Similarly the vector $-\mathbf{A}$ has the same magnitude as **A** but points in the reverse direction; its components are $(-A_x, -A_y, -A_z)$.

- Vectors **A** and **B** are added simply by adding their components so that

  $$\mathbf{C} = \mathbf{A} + \mathbf{B} = (A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j} + (A_z + B_z)\mathbf{k}$$

  Hence $C_x = A_x + B_x$ etc. Geometrically one obtains the resultant vector **C** by placing the tail of **B** at the head of **A** (or vice versa) and completing the triangle.