Classwork 8 **- ANSWERS**

1. The distance of closest approach is $s = |\mathbf{V}.\hat{\mathbf{n}}|$ where **V** is any vector between the two paths, and $\hat{\mathbf{n}}$ is the unit normal to both of them, given by the cross product of the two unit direction vectors i.e. $\hat{\mathbf{n}} = \hat{\mathbf{d}}_1 \times \hat{\mathbf{d}}_2$. It follows that $\hat{\mathbf{n}} = \frac{-\mathbf{i}-\mathbf{j}-\mathbf{k}}{\sqrt{3}} \times \frac{4\mathbf{i}+5\mathbf{j}+3\mathbf{k}}{\sqrt{50}} = \frac{2\mathbf{i}-\mathbf{j}-\mathbf{k}}{\sqrt{150}}$. A convenient choice for **V** is $\mathbf{V} = -750(\mathbf{j}+\mathbf{k})$, and from the dot product, one obtains $s = 10\sqrt{150} = 122.5$, a comfortable margin if the units are km.

2.

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\begin{aligned}
e^i &= 1 + \frac{i}{1!} + \frac{i^2}{2!} + \frac{i^3}{3!} + \ldots \\
&= 1 + i - 0.5 - 0.16667i + 0.04167 + 0.00833i - 0.00139 - 0.00020i + \ldots \\
&= (1 - 0.5 + 0.04167 - 0.00139 + \ldots) + i(1 - 0.16667 + 0.00833 - 0.00020 + \ldots) \\
&= 0.54028 + 0.84146i
\end{aligned}
$$

The modulus and argument both come out as 1 ….. but, since $e^{i\theta} = \cos\theta + i\sin\theta$, you have been working out cos(1) and sin(1), so the answer should be no surprise at all!

3. (i) The eigenvalues are 4 and –1, and the respective (un-normalised) eigenvectors are (1, 1) and (3, –2).

(ii) The characteristic equation is $\lambda^3 - 5\lambda^2 - \lambda + 5 = (\lambda - 5)(\lambda^2 - 1) = 0$, so the eigenvalues are 5, 1, and –1. The respective eigenvectors are (1, 1, 0), (0, 0, 1) and (1, –1, 0).

(iii) The characteristic equation is $\lambda^3 - 4\lambda^2 - \lambda + 4 = (\lambda - 1)(\lambda + 1)(\lambda - 4) = 0$, so the eigenvalues are 4, 1, and –1. The respective eigenvectors are (1, 1, 1), (1, 1, -2) and (1, –1, 0).