Classwork 7 - ANSWERS

1. $X = ax + by, \quad Y = cx + dy.$ The origin is transformed into itself.

2. (i) $\mathbf{r}_\mathrm{A} = \begin{pmatrix} u \\ v \end{pmatrix}, \quad \mathbf{r}_\mathrm{B} = \begin{pmatrix} u+s \\ v \end{pmatrix}, \quad \mathbf{r}_\mathrm{C} = \begin{pmatrix} u+s \\ v+s \end{pmatrix}, \quad \mathbf{r}_\mathrm{D} = \begin{pmatrix} u \\ v+s \end{pmatrix}.$

   (ii) $\overrightarrow{AB} = s\mathbf{i}, \quad \overrightarrow{DC} = s\mathbf{i} \quad \overrightarrow{AD} = s\mathbf{j}, \quad \overrightarrow{BC} = s\mathbf{j}$.

3. If $\mathbf{r} = \mathbf{r}_0 + \lambda\mathbf{d}$, then $\mathbf{R} = \mathbf{R}_0 + \lambda\mathbf{D}$ where $\mathbf{D} = \mathbf{Md}$.

   $\mathbf{r}_E = \begin{pmatrix} au+bv \\ cu+dv \end{pmatrix}, \quad \mathbf{r}_F = \begin{pmatrix} a(u+s)+bv \\ c(u+s)+dv \end{pmatrix}, \quad \mathbf{r}_G = \begin{pmatrix} a(u+s)+b(v+s) \\ c(u+s)+d(v+s) \end{pmatrix}, \quad \mathbf{r}_H = \begin{pmatrix} au+b(v+s) \\ cu+d(v+s) \end{pmatrix}.$

   $\overrightarrow{EF} = \begin{pmatrix} as \\ cs \end{pmatrix}, \quad \overrightarrow{HG} = \begin{pmatrix} as \\ cs \end{pmatrix}, \quad \overrightarrow{EH} = \begin{pmatrix} bs \\ ds \end{pmatrix}, \quad \overrightarrow{FG} = \begin{pmatrix} bs \\ ds \end{pmatrix}.$

4. $\mathbf{r}_E = \begin{pmatrix} -5 \\ -6 \end{pmatrix}, \quad \mathbf{r}_F = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \quad \mathbf{r}_G = \begin{pmatrix} 10 \\ 12 \end{pmatrix}, \quad \mathbf{r}_H = \begin{pmatrix} 1 \\ 6 \end{pmatrix}.$

   $\overrightarrow{EF} = \begin{pmatrix} 9 \\ 6 \end{pmatrix} = \overrightarrow{HG}, \quad \overrightarrow{EH} = \begin{pmatrix} 6 \\ 12 \end{pmatrix} = \overrightarrow{FG}.$

5. Since $\overrightarrow{EF} = as\mathbf{i} + cs\mathbf{j}$ and $\overrightarrow{EH} = bs\mathbf{i} + ds\mathbf{j}$, the area is

   $$\left\| \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ as & cs & 0 \\ bs & ds & 0 \end{vmatrix} \right\| = \left\| s^2(ad-bc)\mathbf{k} \right\| = s^2 \left|(ad-bc)\right|.$$

   Hence, since the original area was s^2, the area scale factor is $|ad-bc| = |\det \mathbf{M}| = 8$ in the case of question 4.

   Note that, in the equation for the area, the outer bars mean the vector magnitude, and the inner bars the determinant.

6. Yes, because any shape can be considered to be an assembly of small squares.

7. (i) The vectors are $\mathbf{A} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}, \quad \mathbf{B} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}, \quad \mathbf{C} = c_1\mathbf{i} + c_2\mathbf{j} + c_3\mathbf{k}.$

   (ii) The volume is $\left|(\mathbf{A}_1 \times \mathbf{A}_2).\mathbf{A}_3\right| = \left\| \begin{matrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{matrix} \right\| = |\det \mathbf{T}|$ as before.

   (iii) Yes, because any solid can be considered to be an assembly of small cubes.