***Classwork 6* – ANSWERS** *(corrected)*

(a) (i) $x = A\cos\omega t$. (ii) If $\tilde{\omega} = i\gamma$, then $x = Ae^{-\gamma t}$ which represents exponential decay if γ is positive.

(b) (i) $x = A\cos(\omega t - \pi/2) = A\sin\omega t$. (ii) $x = A\cos(\omega t \pm \pi) = -A\cos\omega t$.

(c) (i) $x = Ae^{-\gamma t}\cos\omega t$, so $x = A$ at $t = 0$. (ii) $T = 2\pi/\omega$, (iii) $e^{-\gamma\tau} = 0.5$ so $\tau = \dfrac{\log_e 2}{\gamma}$.

(d) $m(-\omega_0^2\tilde{x}) = -k\tilde{x}$ so $\omega_0 = \sqrt{k/m}$.

(e) $\tilde{v} = \dfrac{d\tilde{x}}{dt} = i\omega_0\tilde{x} = i\omega_0\tilde{A}e^{i\omega_0 t}$.

(f) $x = \mathrm{Re}\{\tilde{x}\} = \mathrm{Re}\{Ae^{i\omega_0 t}\} = A\cos\omega_0 t$; $v = \mathrm{Re}\{\tilde{v}\} = \mathrm{Re}\{i\omega_0 Ae^{i\omega_0 t}\} = -\omega_0 A\sin\omega_0 t$.

(g) (i) $\omega_0 = 5$ rad/s; (ii) $T = 2\pi/5 = 1.26$s;

(iii) At $t = 0$, $x = 0.1$ m, $v = 0$ m/s.

At $t = 0.2$ s, $x = 0.1\cos(1.0) = 0.054$ m, $v = -0.5\sin(1.0) = -0.421$ m/s.

At $t = 0.4$ s, $x = 0.1\cos(2.0) = -0.042$ m, $v = -0.5\sin(2.0) = -0.455$ m/s.

(h) $x = \mathrm{Re}\{\tilde{x}\} = \mathrm{Re}\{Ae^{i(\omega_0 t + \pi/2)}\} = A\cos(\omega_0 t + \pi/2) = -A\sin\omega_0 t$;

$v = \mathrm{Re}\{\tilde{v}\} = \mathrm{Re}\{i\omega_0 Ae^{i(\omega_0 t + \pi/2)}\} = -\omega_0 A\sin(\omega_0 t + \pi/2) = -\omega_0 A\cos\omega_0 t$.

(i) $m\tilde{\omega}^2 - i\rho\tilde{\omega} - k = 0$ leads to $\tilde{\omega} = \dfrac{i\rho}{2m} \pm \sqrt{\dfrac{k}{m} - \dfrac{\rho^2}{4m^2}}$.