Classwork 3
Systematic Elimination

This classwork is about solving simultaneous linear equations using a systematic technique for eliminating the unknowns.

First consider the set of equations

(1) $2x - y + 3z = 9$

(2) $x - y + 4z = 10$

(3) $3x + y + 2z = 6$

1. Use Cramer's rule (reproduced from Fact Sheet F at the end) to find *x*. You'll find it's a real chore!

2. Now follow these steps to solve the problem.
   (a) Exchange eqs. (1) and (2)
   (b) Subtract 3× the new eq.(1) from eq.(3) to eliminate *x* from this equation.
   (b) Similarly subtract 2× eq.(1) from eq.(2) to eliminate *x* from this equation too.
   (d) Subtract 4× the modified eq.(2) from eq.(3) to eliminate *y* from the equation.
   (e) Obtain *z* from the final version of eq.(3), *y* from the final version of eq.(2), and *x* from the final version of eq.(1).
   (f) Substitute the values of *x*, *y* and *z* back into the original equations to verify that it all checks out.

3. Now try to solve this set without the clues:

$$x + 2y + z = 7$$

$$-2x + 3y - z = -5$$

$$3x + 12y - 6z = 9$$

Once again, devise a strategy to eliminate *x* from the second equation and both *x* and *y* from the third. There's a hint at the end

4. If you're still on board, have a shot at this 4×4 system:

$$w + 2x + y + 3z = 18$$

$$2w + 4x + 6y + z = -3$$

$$w + 3x \qquad + 5z = 24$$

$$3w + 5x + 2y + 4z = 40$$

5. There's an obvious problem with this set of equations. Can you see what it is?

$$2x - y + 3z = 9$$

$$x - y + 4z = 10$$

$$6x - 3y + 9z = 27$$

6. And what about this set?

$$x + 3y - z = 6$$

$$8x + 9y + 4z = 21$$

$$2x + y + 2z = 3$$

Cramer's Rule

For the 3×3 system

$$\begin{aligned} a_1x + b_1y + c_1z &= k_1 \\ a_2x + b_2y + c_2z &= k_2 \\ a_3x + b_3y + c_3z &= k_3 \end{aligned}$$

the solution for x is $x = \dfrac{\begin{vmatrix} k_1 & b_1 & c_1 \\ k_2 & b_2 & c_2 \\ k_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} = \dfrac{k_1b_2c_3 - k_1b_3c_2 - k_2b_1c_3 + k_2b_3c_1 + k_3b_1c_2 - k_3b_2c_1}{a_1b_2c_3 - a_1b_3c_2 - a_2b_1c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c_1}$

Hint for question 3

Start by subtracting the first equation from the third, after removing the common factor of 3 from the latter.

Rules of the game

None of the following operations changes the solution of a set of linear equations:

(i) Changing the order of the equations.

(ii) Multiplying all terms in an equation by the same constant.

(iii) Adding a multiple of any equation to any other equation. The multiple can be negative (so addition includes subtraction), and it need not be an integer multiple (so a fraction is OK).

The strategy is to leave only z in the last equation, only y and z in the next last, and so on.