1st Year – Maths Techniques II (Algebra) **CW2: 05/11/2004**

Classwork 2 – ANSWERS

1. For PH01 one obtains $x_1 = -20 + \lambda$, $y_1 = 20 + 2\lambda$, $z_1 = 5$; the result follows immediately. The analogous equations for PH02 are $x_2 = 5 - \mu$, $y_2 = 5 + \mu$, $z_2 = 7 - \dfrac{23\mu}{260}$.

   Only the *x* and *y* coordinates count at this stage, and the results quoted follow immediately by eliminating λ and μ.

2. Setting $x_1 = x_2 = x$ and $y_1 = y_2 = y$, and solving the simultaneous equations yields $x = -50/3 = -16.7$ km and $y = 80/3 \approx 26.7$ km.

3. $\cos\theta = \dfrac{\mathbf{d}_1.\mathbf{d}_2{}'}{|\mathbf{d}_1||\mathbf{d}_2{}'|}$ where $\mathbf{d}_2{}' = -\mathbf{i} + \mathbf{j}$. One obtains $\cos\theta = \dfrac{1}{\sqrt{10}}$ and $\theta = 1.25\,\text{rad} = 71.6°$.

4. The unit normals for the two flight paths are $\hat{\mathbf{n}}_1 = -\dfrac{2}{\sqrt{5}}\mathbf{i} + \dfrac{1}{\sqrt{5}}\mathbf{j}$ and $\hat{\mathbf{n}}_2 = \dfrac{1}{\sqrt{2}}\mathbf{i} + \dfrac{1}{\sqrt{2}}\mathbf{j}$. Taking the dot product of each unit normal with <u>*any*</u> position vector on the respective paths yields $p_1 = \dfrac{60}{\sqrt{5}} = 26.8$ km, and $p_2 = 5\sqrt{2} = 7.07$ km. The latter result should be obvious from your sketch map.

5. $\tan\delta = \dfrac{23}{260\sqrt{2}}$ leads to $\theta = 3.6°$.

6. Since $\mu = 5 - x$, $x = -50/3$, and $z = 7 - \dfrac{23\mu}{260}$, it follows that $\mu = 65/3$ and $z_2 = 7 - \dfrac{23\times 65}{260\times 3} = 5.083$. The separation is therefore about 83 metres.

7. The controller should either tell PH01 to drop to a lower altitude, or tell PH02 to delay its descent.

8. The altitude of PH01 is 5 km, so the distance in this case is $\sqrt{p_1^2 + 5^2} = \sqrt{745} = 27.3$ km. For PH02, the direction of the perpendicular is affected, albeit very slightly, by the downward slant of the flight path. However, if one ignores this subtlety, an excellent approximation can be obtained by assuming that the (*x*, *y*) coordinates of the point of closest approach are the same as in 2D. Since the altitude is 7 km at this point, one obtains $\sqrt{p_2^2 + 7^2} = \sqrt{99} = 9.95$ km for the distance. (By using a vector product formula to find the perpendicular distance, I made the exact answer $\sqrt{98.81} = 9.94$ km.)

   The angle is changed but, again, only marginally. In the new case, $\cos\theta = \dfrac{1}{\sqrt{5}\sqrt{2 + \left(\dfrac{23}{260}\right)^2}}$ which leads to $\theta = 71.602°$, compared with 71.565° previously.