Classwork 2
Air Traffic Control

- Flight PH01 is flying level at an altitude of 5 km following the track $\mathbf{r}_1 = \mathbf{r}_{10} + \lambda \mathbf{d}_1$ where $\mathbf{r}_{10} = -20\mathbf{i} + 20\mathbf{j}$ and $\mathbf{d}_1 = \mathbf{i} + 2\mathbf{j}$.
- Flight PH02 is descending on the track $\mathbf{r}_2 = \mathbf{r}_{20} + \mu \mathbf{d}_2$ where $\mathbf{r}_{20} = 5\mathbf{i} + 5\mathbf{j} + 7\mathbf{k}$ and $\mathbf{d}_2 = -\mathbf{i} + \mathbf{j} - \frac{23}{260}\mathbf{k}$.

  All distances are in km. The *x* coordinate is east, *y* is north, and *z* is altitude. The origin is a radio beacon.

Ignore the *z* dimension (altitude) to begin with, and just consider the aircraft flight paths as lines on a map or chart.

1. Show that the path of flight PH01 is given by

   $$x_1 + 20 = \frac{y_1 - 20}{2}$$

   and that of flight PH02 is given by

   $$x_2 - 5 = -(y_2 - 5)$$

2. Plot the flight paths on a sketch map, and find the coordinates of the point where they cross.
3. Find the angle between the two flight paths on the map.
4. Find the distance from the beacon to the closest point on each flight path.

Now include altitude and consider the paths of the aircraft in 3D.

5. Find the angle of descent of flight PH02.
6. Find the vertical separation of the two flight paths at the point where they cross.
7. If you were the air traffic controller and the planes were expected to arrive at the crossing point at roughly the same time, what would you advise?
8. (Harder) Find the nearest distance of each aircraft to the beacon. This is similar to question 4, but altitude is now involved. Note that to obtain a fully accurate result for PH02 would involve a complicated calculation, but you can get a very good approximation quite easily.

   Is the angle between the two flight paths in 3D the same as the answer to question 3?