1st Year – Maths Techniques II (Algebra) **01/11/2004**

Classwork 1 – *Square Root of a Complex Number*

If a complex is raised to a power (even a complex power), it produces another complex number. In this classwork, we explore square roots of complex numbers; imaginary powers turn up in a later classwork. The roots of complex numbers will be covered more systematically later in the course.

1. Consider $w = z^2$ where $z = x + iy$ is a general complex number. Write down, in terms of x and y):

   (i) w (ii) $\mathrm{Re}\{w\}$ (iii) $\mathrm{Im}\{w\}$ (iv) $|z|$ (v) $|w|$

   What is the relationship between $|w|$ and $|z|$?

In the following questions, consider the case where $w = z^2 = 2(1 + i\sqrt{3})$.

2. Find $|w|$ and $|z|$.

3. (i) obtain equations for *x* and *y*, the real and imaginary parts of *z*.

   (ii) by eliminating *y* from these equations, show that

   $x^4 - 2x^2 - 3 = 0$

   (iii) How many roots does this equation have (i.e. how many values of *x* satisfy it)?

   (iv) Are all the roots appropriate in this case? If not, state how many ***are*** appropriate and find the corresponding values of *y*.

   (v) Each (*x*, y) pair defines a complex number *z*. Check that the modulus of each *z* has the value predicted in question 2, and that the correct value of *w* is recovered if the square is taken.

   (vi) Plot *w* and its roots on an Argand diagram.

   (vii) ***Harder ...*** Express *w* and *z* in complex exponential form, finding the arguments of each (in radians).

   (viii) Repeat parts (i)-(vii) in the case where $w = z^2 = 2(-1 + i\sqrt{3})$.

   Spend a little time looking at any roots you discarded in part (iv) and see if you can find anything interesting about them!