**Maths Techniques II: Algebra**
**Lecturer: Professor Geoff New**

## Aims

To give students a basic understanding of vectors, complex numbers, linear equations, determinants, matrices, linear transformations, eigenvalues, eigenvectors, and associated topics.

## Objectives

- Know how to convert from 2D Cartesian coordinates to plane polar coordinates, and vice versa.
- Know what is meant by a right-handed set of 3D Cartesian coordinates.
- Know the meaning of the terms: vector, scalar, position vector, vector component etc.
- Know what is meant by a unit vector, and how the special unit vectors **i**, **j**, and **k** are defined.
- Be able to find the unit vector in the direction of a given vector, and know how to specify a direction using direction cosines.
- Be able to calculate the magnitude of a vector given its components.
- Know how to add and subtract vectors both geometrically and in component form.
- Know how to find the vector displacement from one point to another, given the position vectors or coordinates of the points.
- Know how to multiply a vector by a scalar.
- Understand, and be able to work out the scalar (dot) and vector (cross) products of two vectors.
- Understand different forms of the equation of a straight line, including the form using vectors.
- Know how to use the scalar product to find the angle between two vectors or two lines.
- Be aware that area can be represented as a vector quantity.
- Understand how to use the vector product to find the area of a parallelogram and the area of a triangle.
- Be able to differentiate a time-varying vector, and know how to work out the derivative of scalar and vector products.
- Know what is meant by the triple scalar product and the triple vector product of three vectors, understand their properties, and be able to use the triple scalar product to find the volume of a parallelepiped and the volume of a tetrahedron.
- Know that any vector lying in a given plane can be written as a linear combination of two other non-parallel vectors in the plane.
- Understand the concept of three linearly independent vectors geometrically, and be able to use the triple scalar product to determine if three vectors are linearly independent.
- Know the meaning of the symbol $i$ in complex number arithmetic.
- Know what is meant by the real part, the imaginary part, the modulus and the argument of a complex number, and how these attributes are related to each other.
- Know how a complex number can be represented on an Argand diagram, and be able to plot a complex number in the complex plane.
- Understand what is meant by the complex conjugate of a complex number.
- Know how to express complex numbers in exponential form.
- Know how to multiply and divide two complex numbers both in rectangular and exponential form.

- Understand how to handle integer multiples of $2\pi$ radians in the argument of a complex number.
- Be familiar with formulae linking complex exponentials and trigonometric functions.
- Understand the connection between trigonometric and hyperbolic functions.
- Be able to evaluate some elementary functions of complex numbers (exponentials, logs, powers, sines, cosines etc.)
- Be able to find the nth roots of an arbitrary complex number including unity.
- Know the number of roots of an nth degree polynomial equation, and be able to prove that the complex roots of such an equation with real coefficients occur in conjugate pairs.
- Have some appreciation of the use of complex numbers in mathematical physics.
- Know what is meant by a set of linear equations.
- Know what matrices and determinants are.
- Understand general matrix and determinant terminology (row, column, element, order, diagonal, transpose, minor, cofactor etc.)
- Understand the different suffix notations for identifying matrix/determinant elements.
- Understand the basic properties of determinants and be able to evaluate $2\times2$, $3\times3$ and $4\times4$ determinants.
- Be able to solve simultaneous linear equations using Cramer's rule or by systematic elimination.
- Know how to discover if a given set of inhomogeneous equations has a unique solution or a given set of homogeneous equations a non-trivial solution. Understand the associated geometrical interpretations.
- Be able to recognise special types of matrices: row, column, square, diagonal, identity, orthogonal, etc.
- Understand the conditions under which two matrices can be added, subtracted and multiplied, and be able to perform these operations.
- Know that the components of a vector can be contained in row and column matrices, and how to represent the scalar product in matrix form.
- Know what is meant by a singular matrix and be able to determine if a matrix is singular.
- Know how to rotate coordinates in 2D and to represent the process using $2\times2$ matrices.
- Know the basic properties of more general linear transformations expressed as matrices.
- Understand the relationship between matrix multiplication and successive linear transformations.
- Know how to find the area scale factor for a linear transformation in 2D, and know what is special about a linear transformation in 2D for which the matrix is singular.
- Know how to find both the adjoint and the inverse of $2\times2$ and $3\times3$ matrices, and be able to use matrix inversion to solve sets of two or three linear equations.
- Know the basic form of a matrix eigenvalue equation and what the terms eigenvalue and eigenvector mean.
- Be able to write down the characteristic equation of a $2\times2$ and a $3\times3$ matrix.
- Be able to find the eigenvalues and eigenvectors of matrices in simple cases.
- Know the meaning of the term orthonormal vector and be able to normalise eigenvectors.
- Understand the meaning of eigenvalues and eigenvectors in the context of linear transformations in 2D.