ANSWERS to Lecture 13 problems

1. (a) Determinant $\neq 0$; no non-trivial solution exists.

   (b) Determinant $\neq 0$; no non-trivial solution exists.

   (c) Determinant = 0; the equations are scaled versions of $2x + y = 0$, which defines the line solution.

   (d) Determinant = 0; the equations are scaled versions of $7x - 6y = 0$, which defines the line solution.

2. (a) Determinant = 0; so a non-trivial solution exists.

   (b) Determinant = 63; no non-trivial solution exists.

   (c) Determinant = 0 by inspection (col 2 = –2 × col 4), so a non-trivial solution exists.

3. (a) $-145\mathbf{i} - 34\mathbf{j} + 116\mathbf{k}$ (b) $-49\mathbf{i} + 14\mathbf{j} - 28\mathbf{k}$

4. (a) If you attempt to find *p* and *q*, you discover that the equations are inconsistent.

   (b) Linear dependence implies that the vectors are coplanar; linear independence implies that they are *not* coplanar. The determinant formed from the components of the vectors is the triple scalar product, and represents the volume of the parallelepiped based on the three vectors. This is zero when the vectors are coplanar i.e. when the vectors are linearly dependent, and non-zero otherwise.

   (c) The determinant $\begin{vmatrix} 2+\alpha & 1 & -3 \\ 7 & -2 & 4 \\ 4 & 0 & 5 \end{vmatrix} = 63 + 10\alpha$, so setting $\alpha = -6.3$ makes it zero.

5. $$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) + \mathbf{B}\times(\mathbf{C}\times\mathbf{A}) + \mathbf{C}\times(\mathbf{A}\times\mathbf{B})$$
$$= (\mathbf{A}.\mathbf{C})\mathbf{B} - (\mathbf{A}.\mathbf{B})\mathbf{C} + (\mathbf{B}.\mathbf{A})\mathbf{C} - (\mathbf{B}.\mathbf{C})\mathbf{A} + (\mathbf{C}.\mathbf{B})\mathbf{A} - (\mathbf{C}.\mathbf{A})\mathbf{B} = 0$$