Homogeneous Equations, Triple Products, Linear Independence

1. Determine which of following pairs of homogeneous equations have a non-trivial solution and, in those cases that do, find the equation of the line that represents the solution

   (a) $3x+5y=0, \quad 2x+4y=0.$

   (b) $3x-5y=0, \quad 7x+2y=0.$

   (c) $6x+3y=0, \quad 4x+2y=0.$

   (d) $1.4x-1.2y=0, \quad -2.1x+1.8y=0.$

2. Determine which of following sets of homogeneous equations have a non-trivial solution,

   (a) $8x+y+8z=0, \quad 6x+4y+4z=0, \quad 5x-y+6z=0.$

   (b) $5p+2q+2r=0, \quad p-q+4r=0, \quad 7p+r=0.$

   (c) $12x_1-16x_2+2x_3+8x_4=0, \quad -6x_1+6x_2+14x_3-3x_4=0,$
   $10x_1+10x_2-7x_3-5x_4=0, \quad 11x_1-18x_2+2x_3+9x_4=0.$

3. If $\mathbf{A} = 2\mathbf{i}+\mathbf{j}-3\mathbf{k}$, $\mathbf{B} = 7\mathbf{i}-2\mathbf{j}+4\mathbf{k}$, and $\mathbf{C} = 4\mathbf{i}+5\mathbf{k}$, find the following triple vector products:

   (a) $(\mathbf{A}\times\mathbf{B})\times\mathbf{C}$, (b) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C})$.

4. Three vectors **A**, **B**, and **C** are "linearly dependent" if it is possible to write one as a linear combination of the other two, i.e. $\mathbf{A} = p\mathbf{B}+q\mathbf{C}$. Three vectors are "linearly independent" if it is <u>*not*</u> possible to do this.

   (a) Show that the vectors defined in question 3 are linearly independent.

   (b) What does linear dependence or independence imply about the determinant formed from the components of the three vectors?

   (c) The vector $\alpha\mathbf{i}$ is now added to **A**. What value of α makes the three vectors linearly dependent?

5. Show that for any three vectors **A**, **B**, and **C**

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C})+\mathbf{B}\times(\mathbf{C}\times\mathbf{A})+\mathbf{C}\times(\mathbf{A}\times\mathbf{B})=0$$