ANSWERS to Lecture 12 problems

1. (a) non-singular; determinant = 4; inverse = $\begin{pmatrix} 1 & -\frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$ (b) singular

   (c) non-singular; determinant = 1; inverse = $\begin{pmatrix} 5 & -3 \\ -3 & 2 \end{pmatrix}$.

2 (a) Singular: column 2 is 2 × column 3.

   (b) The determinant is $-7\times(6-30)=168$, so the matrix is non-singular.

   (c) The matrix is singular: row 3 is (3 × row 1 + 2 × row 2).

   (d) The determinant is easily evaluated as –8, so the matrix is non-singular.

   (e) Singular: rows 1 and 3 are the same.

   (f) Since this matrix is not square, it has no determinant, so the issue of singularity does not arise. However, the term "singular" is sometimes applied to *<u>any</u>* matrix that has no inverse, in which case this matrix, which certainly has no inverse, would count as singular!

3. The matrix of the coefficients is $\mathbf{A}=\begin{pmatrix} -1 & 2 & 3 \\ 2 & 1 & -4 \\ -1 & -2 & 1 \end{pmatrix}$.

   (a) det **A** = 2 (b) $\mathbf{C}=\begin{pmatrix} -7 & 2 & -3 \\ -8 & 2 & -4 \\ -11 & 2 & -5 \end{pmatrix}$ (c) $\text{adj}\,\mathbf{A}=\mathbf{C}^{\mathrm{T}}=\begin{pmatrix} -7 & -8 & -11 \\ 2 & 2 & 2 \\ -3 & -4 & -5 \end{pmatrix}$

   (d) $\mathbf{A}^{-1}=\dfrac{\text{adj}\,\mathbf{A}}{\det\mathbf{A}}=\begin{pmatrix} -\frac{7}{2} & -4 & -\frac{11}{2} \\ 1 & 1 & 1 \\ -\frac{3}{2} & -2 & -\frac{5}{2} \end{pmatrix}$ (e) $\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} -\frac{7}{2}k_1-4k_2-\frac{11}{2}k_3 \\ k_1+k_2+k_3 \\ -\frac{3}{2}k_1-2k_2-\frac{5}{2}k_3 \end{pmatrix}$

4. (a) $\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} -\frac{7}{2}-4+\frac{11}{2} \\ 1+1-1 \\ -\frac{3}{2}-2+\frac{5}{2} \end{pmatrix}=\begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}$ (b) $\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} -\frac{35}{2}+32 \\ 5-8 \\ -\frac{15}{2}+16 \end{pmatrix}=\begin{pmatrix} \frac{29}{2} \\ -3 \\ \frac{17}{2} \end{pmatrix}$ (c) $\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} -54 \\ 12 \\ -25 \end{pmatrix}$

5. You can transpose the matrix and use the formula at the top of the second side of Fact Sheet H, namely $D_4=a_{11}C_{11}+a_{21}C_{21}+a_{31}C_{31}+a_{41}C_{41}$; this formula is based on an expansion down the first column of the matrix. However, you can avoid the transpose by expanding along the first *<u>row</u>* of the matrix, which yields the entirely equivalent formula $D_4=a_{11}C_{11}+a_{12}C_{12}+a_{13}C_{13}+a_{14}C_{14}$. Whichever way you do it, $D_4=6$.