Fact Sheet K – Triple Products

Triple Scalar Product

- The triple scalar product, so called because three vectors are involved and the answer is a scalar quantity, is defined as

  $(\mathbf{A}\times\mathbf{B}).\mathbf{C} = \mathbf{C}.(\mathbf{A}\times\mathbf{B})$.

- In component form it reads

  $$(\mathbf{A}\times\mathbf{B}).\mathbf{C} = C_x(A_yB_z - A_zB_y) + C_y(A_zB_x - A_xB_z) + C_z(A_xB_y - A_yB_x)$$

  $$= \begin{vmatrix} C_x & A_x & B_x \\ C_y & A_y & B_y \\ C_z & A_z & B_z \end{vmatrix} = \begin{vmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ A_z & B_z & C_z \end{vmatrix} = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}.$$

- The magnitude of the triple scalar product is the volume of a parallelepiped whose sides are the vectors **A**, **B**, and **C**.

Triple Vector Product

- The triple vector product, so called because three vectors are involved and the result is a vector quantity, is defined as

  $\mathbf{T} = \mathbf{C}\times(\mathbf{A}\times\mathbf{B})$

- Bearing in mind that **A**×**B** is perpendicular to the plane of **A** and **B**, and that **T** is perpendicular to the plane of **C** and **A**×**B,** one can deduce that **T** lies in the plane of **A** and **B**. It is therefore reasonable that

  $\mathbf{T} = p\mathbf{A} + q\mathbf{B}$.

  It can be shown (but the proofs are messy) that $p = \mathbf{C.B}$ that $q = -\mathbf{C.A}$, so the full result is

  $\mathbf{C}\times(\mathbf{A}\times\mathbf{B}) = (\mathbf{C.B})\mathbf{A} - (\mathbf{C.A})\mathbf{B}$

  or equivalently

  $(\mathbf{A}\times\mathbf{B})\times\mathbf{C} = (\mathbf{C.A})\mathbf{B} - (\mathbf{C.B})\mathbf{A}$