Fact Sheet E– Vector Multiplication

There are two types of vector × vector multiplication:

- The *scalar product*, so called because the results is a *scalar* quantity (and also called the *dot product* because a dot is the symbol used) is defined as

  $$\mathbf{A.B} = |\mathbf{A}||\mathbf{B}|\cos\theta$$

  where $|\mathbf{A}|$ and $|\mathbf{B}|$ are the magnitudes of the two vectors and θ is the angle between the vectors.

  In terms of components,

  $$\mathbf{A.B} = A_x B_x + A_y B_y + A_z B_z$$

- The vector *product*, so called because the result is a *vector* quantity (and also called the *cross product* because a cross is the symbol commonly used) is defined as

  $$\mathbf{A} \times \mathbf{B} = |\mathbf{A}||\mathbf{B}|\sin\theta\ \hat{\mathbf{n}}$$

  where $\hat{\mathbf{n}}$ is a unit vector perpendicular to the plane of **A** and **B**, directed such that **A**, **B**, and $\hat{\mathbf{n}}$ form a right-handed system.

  In terms of components

  $$\mathbf{A} \times \mathbf{B} = \mathbf{i}(A_y B_z - A_z B_y) + \mathbf{j}(A_z B_x - A_x B_z) + \mathbf{k}(A_x B_y - A_y B_x)$$

  In determinant form (see later in the course), the vector product can be written

  $$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

- From the definitions, it is easy to see that, whereas the scalar product is commutative, the vector product is non-commutative i.e.

  $\mathbf{A.B} = \mathbf{B.A}$ but $\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$

- Some books use the symbolism $\mathbf{A} \wedge \mathbf{B}$ for the vector product.

- The unit vectors **i**, **j**, and **k** obey the following fairly obvious scalar product rules:

  $$\mathbf{i.i} = \mathbf{j.j} = \mathbf{k.k} = 1$$

  but the scalar product of all unlike pairs is zero, i.e.

  $$\mathbf{i.j} = \mathbf{j.k} = \mathbf{k.i} = 0$$

- The vector product of any vector with itself is always zero so

  $\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = 0$.

  Provided a right-handed system of axes is in use

  $\mathbf{i} \times \mathbf{j} = \mathbf{k} \quad \mathbf{j} \times \mathbf{k} = \mathbf{i} \quad \mathbf{k} \times \mathbf{i} = \mathbf{j}$

  $\mathbf{j} \times \mathbf{i} = -\mathbf{k} \quad \mathbf{k} \times \mathbf{j} = -\mathbf{i} \quad \mathbf{i} \times \mathbf{k} = -\mathbf{j}$

  Note that the sign is positive when the three unit vectors are in *cyclic order* (i j k i j k ....), and negative otherwise (see Fact Sheet B).

- Note that right-handedness occurs both in the definition of the vector product, and in the definition of a right-handed set of *xyz* axes. Failure to adhere strictly to these rules will lead to sign errors.