***Classwork 5* – ANSWERS** *(corrected)*

(a) The magnitude of **u** ($|\mathbf{u}| = \sqrt{\mathbf{u.u}}$) is the root of the sum of the squares of the components, so $u_x^2 + u_y^2 = v_x^2 + v_y^2 = 1$ in this case. The two vectors are orthogonal if $\mathbf{u.v} = 0$, so $u_x v_x + u_y v_y = 0$. In matrix form, the conditions are $\mathbf{v}^\mathrm{T}\mathbf{v} = \mathbf{u}^\mathrm{T}\mathbf{u} = 1$ and $\mathbf{v}^\mathrm{T}\mathbf{u} = \mathbf{u}^\mathrm{T}\mathbf{v} = 0$.

(b) $\hat{\mathbf{a}} = \begin{pmatrix} 3/5 \\ -4/5 \end{pmatrix}$ and $\hat{\mathbf{b}} = \begin{pmatrix} \pm 4/5 \\ \pm 3/5 \end{pmatrix}$.

(c) $\begin{pmatrix} u_x & u_y \\ v_x & v_y \end{pmatrix}\begin{pmatrix} u_x & v_x \\ u_y & v_y \end{pmatrix} = \begin{pmatrix} u_x^2 + u_y^2 & u_x v_x + u_y v_y \\ u_x v_x + u_y v_y & v_x^2 + v_y^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ using the conditions in (a).

(d) $\begin{pmatrix} q_x \\ q_y \end{pmatrix} = \begin{pmatrix} cp_x + dp_y \\ ep_x + fp_y \end{pmatrix}$ so $q_x^2 + q_y^2 = (c^2 + e^2)p_x^2 + (d^2 + f^2)p_y^2 + 2p_x p_y(cd + ef) = p_x^2 + p_y^2$.

The conditions for this to be true are $c^2 + e^2 = d^2 + f^2 = 1$ and $cd + ef = 0$, which are exactly the same as the conditions on the elements of **M** in part (c).

(e) $\mathbf{q}^\mathbf{T} = \mathbf{p}^\mathbf{T}\mathbf{N}^\mathbf{T}$.

(f) $\mathbf{q}^\mathrm{T}\mathbf{h} = \mathbf{p}^\mathrm{T}\mathbf{N}^\mathrm{T}\mathbf{N}\mathbf{g}$ using the result of part (e), so $\mathbf{q}^\mathrm{T}\mathbf{h} = \mathbf{p}^\mathrm{T}\mathbf{g}$ if $\mathbf{N}^\mathrm{T}\mathbf{N} = \mathbf{U}$.

(g) Yes.

(h) $\mathbf{L}_1 = \begin{pmatrix} 3/5 & 4/5 \\ -4/5 & 3/5 \end{pmatrix}$ and $\mathbf{L}_2 = \begin{pmatrix} 3/5 & -4/5 \\ -4/5 & -3/5 \end{pmatrix}$. The first represents a clockwise rotation of 53.13°, whereas $\mathbf{L}_2 = \mathbf{L}_1\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, which represents a reflection in the *x*-axis $(y \to -y)$ followed by a 53.13° clockwise rotation.

(i) $\mathbf{t}_1 = \begin{pmatrix} 3/5 & 4/5 \\ -4/5 & 3/5 \end{pmatrix}\begin{pmatrix} -5 \\ 7 \end{pmatrix} = \begin{pmatrix} 13/5 \\ 41/5 \end{pmatrix}$; $\mathbf{t}_2 = \begin{pmatrix} 3/5 & -4/5 \\ -4/5 & -3/5 \end{pmatrix}\begin{pmatrix} -5 \\ 7 \end{pmatrix} = \begin{pmatrix} -43/5 \\ -1/5 \end{pmatrix}$.

The magnitude of all three vectors is $\sqrt{74}$.

It follows that $\mathbf{s.t}_1 = \dfrac{-65 + 287}{5 \times 74} = 0.6$ which corresponds to an angle of 53.13°, and $\mathbf{s.t}_2 = \dfrac{215 - 7}{5 \times 74} = 0.5622$ which corresponds to an angle of 55.79°.

Relative to the negative *x*-axis, the vector **s** lies at 54.46° clockwise, the vector $\mathbf{t}_1$ lies at 107.59° clockwise, and the vector $\mathbf{t}_2$ lies at 1.33° anti-clockwise. The latter can be obtained by reflection of **s** in the *x*-axis to produce $\begin{pmatrix} -5 \\ -7 \end{pmatrix}$ (54.46° anti-clockwise), followed by a 53.13° clockwise rotation.