ANSWERS to Classwork 4 *(corrected)*

(a) If **P** is parallel to $\mathbf{A}\times\mathbf{B}$, it must be perpendicular to the plane defined by **A** and **B**.

(b) V_P represents the volume of a parallelepiped*, in the special case where the two faces in the plane of **A** and **B** are parallelograms, and the other four (perpendicular to the plane of **A** and **B**) are rectangles. (Thus, if one of the parallelograms is the horizontal base of the "box", the four rectangular faces are vertical (of height $|\mathbf{P}|$), and the other parallelogram forms the lid.)

* The most general definition of a parallelepiped is: a solid with six faces, each a parallelogram, and each parallel to the opposite face.

(c) This is the general case in which V is the volume of a parallelepiped whose sides are defined by the three vectors **A**, **B**, and **C**.

(d) If **D** is defined as $\mathbf{D}=\mathbf{A}\times\mathbf{B}$, then $V=|\mathbf{C.D}|=\left|C_xD_x+C_yD_y+C_zD_z\right|$. Substituting $D_x=A_yB_z-A_zB_y$ etc. yields the desired result.

(e) From the result of part (d), it should be clear that

$$V=\begin{vmatrix}C_x & C_y & C_z\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}=\begin{vmatrix}A_x & A_y & A_z\\ B_x & B_y & B_z\\ C_x & C_y & C_z\end{vmatrix},$$

where the slightly more symmetrical second step follows by two row exchanges.

(f) $V=2(2t-s)-(-4)(-t+s)+2(-1+2)=2s+2=4$ for s = 1.

(g) V = 0 when s = –1, irrespective of t.

(h) Substituting **A**, **B** and **C** into $\mathbf{C}=\alpha\mathbf{A}+\beta\mathbf{B}$ shows that the identity also requires s = –1. In this case, $\alpha=-(1+2t)$ and $\beta=-(1+t)$.

(i) The condition s = –1 ensures that **A**, **B**, and **C** are coplanar. Any vector can be written as a superposition of two non-identical vectors in the same plane.