UNIVERSITY OF LONDON

Course:	MC1MF+R
Setter:	Crowdy
Checker:	Luzzato
Editor:	Wu
External:	Cowley
Date:	March 4, 2005

BSc and MSci EXAMINATIONS (MATHEMATICS)
MAY–JUNE 2004

This paper is also taken for the relevant examination for the Associateship.

MC1MF+R Mathematical Methods I

Date: Tuesday, 11th May 2004 Time: 10 am –12 noon

Credit will be given for all questions attempted but extra credit will be given for complete or nearly complete answers.

Calculators may not be used.

Statistical tables will not be available.

Setter's signature	Checker's signature	Editor's signature
....	....	....

1. (a) Find the unique solution of

$$y^{11} = y^{1}\, y$$

With $y(0) = 0,\ y'(0) \ = \ \frac{1}{2}$

(b) Find the general solution of

$$y^{11} \ - 2y' \ + 5y \ = \ e^{x} \, \sin 2x$$

2. (a) Using Euler's substitution or otherwise, find the general solution of,

$$x^{2}\, y^{11} + 3\, x\, y' + y = x^{-1}$$

with $x \ > \ 0.$

(b) Find the general solution to the difference equation

$$n \; U(n) - 2\,(n-1)\, U\,(n-1) + (n-2)\, U\,(n-2) = n$$

3. (a) Find the general solution of the system of ordinary differential equations

$$\begin{aligned} \dot{x} &= 3x - 2y \\ \dot{y} &= 2x - 2y \end{aligned}$$

Also find the uncoupled equation, the equation fro the trajectories and sketch the phase portrait.

(b) Find the general solution to the inhomogeneous system

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} - \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = t \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

. [Hint: note the RHS is polynomial in t with vector coefficient]

4. (a) Is the expression

$$y \sin x\, y\, dx \;+\; x \;\sin\; x\, y\, dy$$

an exact differential? If so, of what function?

(b) Determine the location and nature of the stationery points

$$f(x, y) = x^3 \;+\; y^3 \;-\; 3\, x\, y$$

5. (a) Using the change of variables

$$s = \frac{x}{x^2 + y^2} \qquad , \qquad t = \frac{y}{x^2 + y^2}$$

Show that

$$u_s{}^2 + u_t{}^2 = (u\,x^2 + u\,y^2)(x^2 + y^2)^2$$

where $u = u(x\,,\,y)$.

(b) Find the length of the parametric curve

$$x = \cos^3 t \;\;,\;\; y = \sin^3 t$$

From $t = 0$ to $t = \frac{\pi}{4}$.