Imperial College
London

UNIVERSITY OF LONDON

BSc and MSci EXAMINATIONS (MATHEMATICS)
MAY–JUNE 2004

This paper is also taken for the relevant examination for the Associateship.

M3S10/M4S10 Design of Experiments and Surveys

Date: Monday, 7th June, 2004 Time: 10 am – 12 noon

Credit will be given for all questions attempted but extra credit will be given for complete or nearly complete answers.

Calculators may not be used.

Statistical tables will not be available.

1. **(a)** Explain the term *blocking* in experimental design, and its purpose.

(b) In a cyclic block design the treatments are $\{0, 1, 2, \ldots, t-1\}$ and the initial block is $a_1, a_2, \ldots, a_k$.

Show that for $i \neq j$, treatments i and j occur together in λ_{ij} blocks where λ_{ij} is the number of ways in which $i - j$ can be represented as $a_m - a_n \bmod t$.

(c) By considering the sets $\{1, 7, 11\}$, $\{2, 3, 14\}$ and $\{4, 6, 9\}$, show how to construct a balanced incomplete block design with 19 treatments arranged in blocks of 3 units each.

Justify your construction and state the other parameters of your design. You are not expected to write out the complete design.

(d) Explain, with justification, how you would construct a balanced incomplete block design with 19 treatments arranged in blocks of 16 units each.

2. **(a)** Define

(i) an $n \times n$ *latin square*,

(ii) *a pair of orthogonal* $n \times n$ *latin squares.*

(b) Show that a set of mutually orthogonal $n \times n$ latin squares contains at most $n - 1$ elements.

(c) State (without proof) McNeish's Theorem.

(d) For each statement below indicate whether it is true or false, justifying your answer:

(i) If L is a 4×4 latin square then there exists a 4×4 latin square which is orthogonal to L.

(ii) If n is a multiple of 4, then there exists a pair of orthogonal $n \times n$ latin squares.

3. **(a)** Explain the purpose of fractional replication in a 2^n factorial experiment.

(b) In an industrial experiment there are 5 possible catalysts A, B, C, D and E, each of which may or may not be used in a particular process. The experimenter says:

> "Catalysts A, B and C do not interact with each other or with either D or E. However catalysts D and E are thought to interact with each other and we are interested in exploring this interaction, as well as the main effects of each catalyst. We don't want to test any combinations with all 5 catalysts present, and the fewer combinations with 4 catalysts present the better. By the way, we only have resources to test 8 different combinations".

(i) A decision is made to alias ABE and BCD with the mean. By examining the alias structure of the proposed experiment show it will enable all effects of interest to be estimated unbiasedly.

(ii) Which set of treatment combinations would you use? Justify your answer.

(iii) Construct a partition of these 8 treatment combinations into 2 blocks of 4 each so that no effect of interest is confounded with block effects.

4. **(a)** State (without proof) the General Equivalence Theorem for linear models, explaining fully any assumptions needed and any terms used.

(b) In a particular case, under the assumptions stated in **(a)**, the expected response at the point (x_1, x_2) in the design region

$$\mathcal{X} = \{(x_1, x_2) : 0 \leq x_i \leq 1, \quad i = 1, 2\}$$

is

$$\beta_1 x_1 + \beta_2 x_1 x_2$$

where β_1 and β_2 are unknown parameters.

(i) The design measure ξ attaches weight one-half to each of the points (1, 0) and (1, 1). State whether or nor ξ is D-optimal.
Justify your answer carefully.

(ii) What is a possible argument against using a D-optimal design in practice?

5. **(a)** Define the terms

(i) *simple random sampling without replacement* ,

(ii) *stratified random sampling*.

(b) What is the aim of stratification?

(c) In a survey on alcohol consumption, an estimate of the average number of units of alcohol consumed per week by an Imperial undergraduate is required. Discuss critically how a sample survey to obtain such an estimate could be conducted and indicate any potential difficulties.