1. What is meant by a code C of length n which corrects e errors?

   Define the minimal distance $d(C)$ of C, and state a result linking $d(C)$ and e.

   Now let C be a linear code with check matrix A. Suppose that any $d-1$ columns of A are linearly independent. Prove that $d(C) \geq d$.

   In each of the following cases , say whether or not there is a linear code satisfying the given conditions (give reasoning, stating any standard results you need):

   (a) A linear code of length 11, dimension 5, correcting 2 error.

   (b) A linear code of length 11, dimension 3, correcting 2 errors.

   (c) A linear code of length 11, dimension 4, correcting 2 errors.

2. Define an e-perfect code of length n.

   Define an t-design

   Suppose that C is an e-perfect linear code of length n,

   Prove that the number of codewords in C of weight $2e+1$ os equal to (the quotient of two binomial coefficients $\binom{n}{e+1}$ and $\binom{2e+1}{e}$).
   Let X be the set of n coordinate positions, and for $c\epsilon\ C$ of weight $2e+1$ (i.e. $wt(c) = 2e+1$) let $S(c)$ be the set of positions in which c has a 1. Show that the pair (X, B) where $B = \{S(c)/c\epsilon C,\ wt(c) = 2e+1\}$ form an $(e+1)$ designs with $\lambda_{e+1} = 1$

3. What is meant by a 2-design with parameters (v, k, λ_2)?

   Show that if (X, B) is a 2-design then every point (an element of X) lies in the same number of λ_1 of blocks. Show also that if b is the number of blocks then

   $$b\dot{k} = \nu\dot{\lambda}_1 \quad \text{and} \quad \lambda_1(k-1) = \lambda_2(\nu-1)$$

   Now let (X, B) be a 2-design with parameters $(\nu, 4, 1)$

   (a) Show that $\nu(\nu-1)$ is divisible by 12.

   (b) Show that if (X, B) is symmetric, then $\nu = 13$.

   (c) Give an example of a symmetric 2-design with parameters $(13, 4, 1)$.

4. Let $V = V_n$ be an n-dimensional vector space over field Z_2 of two elements where $n > 3$, and let $1 \leq m \leq n$.

   How many subspaces of dimension m are there in V?

   Let X be the set of non-zero vectors in V.

   For an m-dimensional subspace U in V define $B(U)$ to be the set of non-zero vectors contained in V. Let

$$B = \{B(V)/V \text{ is an } m\text{-dimensional in } v\}$$

Prove that (X, B) is a 2-design and calculate its parameters. Is (X, B) a 3-design?

Let Y be the set of all vectors in V (including the zero vector).

Let $D = \{U+\nu/V$ is an m-dimensional subspace in V and $\nu \varepsilon V\}$ Prove that (Y, D) is a free design and calculate it's parameters. Is Y, D a 4-design?

5. Explain what is meant by a strongly regular graph with parameters (ν, k, a, b).

   For $n \geq 5$, let $T(n)$ be the graph where vertices are the $\binom{n}{2}$ pairs of elements of $\{1, 2, ..., n\}$, with pairs $\{i_2, j_2\}$ joined by one edge and only if $|\{i_1, j_1\} n \{i_2, j_2\}| = 1$. Prove that $T(n)$ is a strongly regular graph, and find its parameters.

   Stating any standard results you require, show that if Γ is a strongly regular graph of valency 12 with 28 vertices, then Γ must have the same parameters (v, k, a, b) as $T(8)$.