M2PM3 PROBLEMS 8, 19.3.2009

Q1. Show that

$$\int_0^{2\pi} \frac{\sin^2\theta}{a + b\cos\theta} d\theta = \frac{2\pi}{b^2}(a - \sqrt{a^2 - b^2}) \qquad (a > b > 0).$$

Q2. Show that

$$\int_0^\infty \frac{dx}{x^4 + a^4} = \frac{\pi}{2\sqrt{2}a^3} \qquad (a > 0).$$

Q3. Show that

$$\sum_{n=-\infty}^\infty \frac{1}{1 + n + n^2} = \frac{2\pi}{\sqrt{3}} \tanh(\pi\sqrt{3}/2).$$

Q4. Evaluate

$$\int_0^\infty \frac{\sin mx}{x} dx$$

for all real m.

NHB