M2P2 Algebra II **Solutions to Sheet 1**

1. FTFFFFTFFTFTTTT

2. (a) is easily checked using the permutations representing the group elements given in leckies.

(b) If $H = \langle \rho \rangle$ then as $D_8 : H| = 2$, the right coset $H\sigma = \{\sigma, \rho\sigma, \rho^2\sigma, \rho^3\sigma\}$ is equal to $D_8 - H$ which is $\{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$.

(c) Using (a) we get $\sigma_1\rho^n = \rho^{-n}\sigma_1$ for all n. Hence $\sigma_i\sigma_j = (\sigma\rho^m)(\sigma\rho^n)$ (for some m, n) $= \sigma\sigma\rho^{-m}\rho^n = \rho^{n-m}$, which is a rotation.

(d) e has order 1; ρ and ρ^3 have order 4; and the other 5 elements ρ^2, σ_i ($i = 1, 2, 3, 4$) have order 2.

(e) The seven cyclic subgroups are $\langle e \rangle$, $\langle \rho \rangle$, $\langle \rho^2 \rangle$, $\langle \sigma_i \rangle$ ($i = 1, 2, 3, 4$).

3. Argue as in Q2.

4. $G(\Pi)$ contains a translation τ moving each D one place to the right, and a reflection σ in the horizontal line bisecting one particular D. Let $g \in G$. Then as shown in lecs (with the symbol F instead of D), for some n, the element $\tau^{-n}g$ fixes each symbol D. The only symmetries of the symbol D are e and σ, so $\tau^{-n}g = e$ or σ, hence $g = \tau^n$ or $\tau^n\sigma$.

Check geometrically that $\tau\sigma = \sigma\tau$. From this it is easy to check that $G(\pi)$ is abelian.

5. (a) Group has size 4, abelian

(b) Size 4, abelian

(c) Group is D_6, non-abelian

(d) Group is D_8, non-abelian

(e) Group is infinite and abelian