Some useful Vector Identities

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$$\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla(\phi) \tag{1}$$

$$\nabla\cdot(\phi\boldsymbol{u}) = \nabla\phi\cdot\boldsymbol{u} + \phi(\nabla\cdot\boldsymbol{u}) \tag{2}$$

$$\nabla\times(\phi\boldsymbol{u}) = \nabla\phi\times\boldsymbol{u} + \phi\,(\nabla\times\boldsymbol{u}) \tag{3}$$

$$\nabla\times(\nabla\phi) = \mathbf{0} \tag{4}$$

$$\nabla\cdot(\nabla\times\boldsymbol{u}) = 0 \tag{5}$$

$$\nabla\cdot(\boldsymbol{u}\times\boldsymbol{v}) = \boldsymbol{v}\cdot(\nabla\times\boldsymbol{u}) - \boldsymbol{u}\cdot(\nabla\times\boldsymbol{v}) \tag{6}$$

$$\nabla\times(\boldsymbol{u}\times\boldsymbol{v}) = (\boldsymbol{v}\cdot\nabla)\boldsymbol{u} - (\boldsymbol{u}\cdot\nabla)\boldsymbol{v} + \boldsymbol{u}(\nabla\cdot\boldsymbol{v}) - \boldsymbol{v}(\nabla\cdot\boldsymbol{u}) \tag{7}$$

$$\nabla(\boldsymbol{u}\cdot\boldsymbol{v}) = (\boldsymbol{v}\cdot\nabla)\boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{v} + \boldsymbol{v}\times(\nabla\times\boldsymbol{u}) + \boldsymbol{u}\times(\nabla\times\boldsymbol{v}) \tag{8}$$

Note that the *divergence* and *curl* of a vector are also written as

$$\nabla\cdot\boldsymbol{u} \equiv \mathrm{div}(\boldsymbol{u})$$

$$\nabla\times\boldsymbol{u} \equiv \mathrm{curl}(\boldsymbol{u})$$