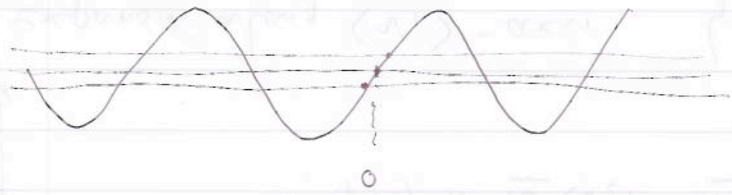
10/2

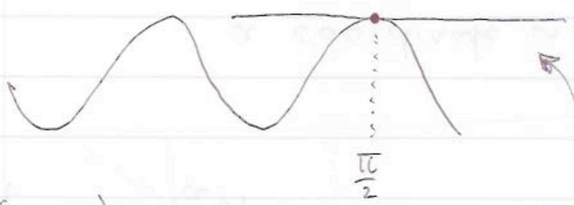
Answers to question in progress test

are on the <u>web</u>!

$F(x,\lambda) = \sin(x) + \lambda$



$D_1F(0,0) = \cos(0) = 1$



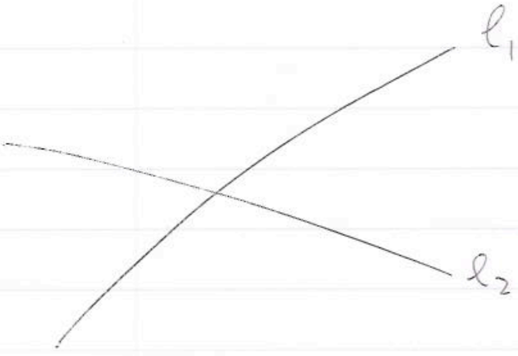
$F(x,1) = \sin(x) - 1$

$F(\frac{\pi}{2},-1) = 0$

$D_1 F(\frac{\pi}{2},1) = \cos(\frac{\pi}{2}) = 0$ noninvertible!

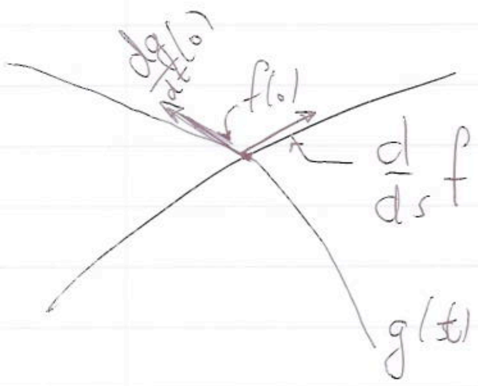
No unique continuation of soln as fn of λ!

Example of bifurcation



change pos of ℓ_1, ℓ_2 a little then unique intersection pt persist (may change pos. slightly) $\qquad 10/2$

What's the appropriate argument in math?



$$f(s) = \begin{pmatrix} f_1(s) \\ f_2(s) \end{pmatrix}$$

$$\frac{d}{ds} f(0) = \begin{pmatrix} \frac{df_1}{ds}(0) \\ \frac{df_2}{ds}(0) \end{pmatrix}$$ tangent to $f(s)$ at $s=0$.

$$Dh_0(0,0) = \begin{pmatrix} \frac{\partial f_1}{\partial s}(0) & -\frac{\partial g_1}{\partial t}(0) \\ \frac{\partial f_2}{\partial s}(0) & -\frac{\partial g_2}{\partial t}(0) \end{pmatrix}$$

$\lambda = 0$

tg vct

$Dh(0,0)$ is invertible <u>iff</u> the columns are lin independent.

$\Updownarrow$

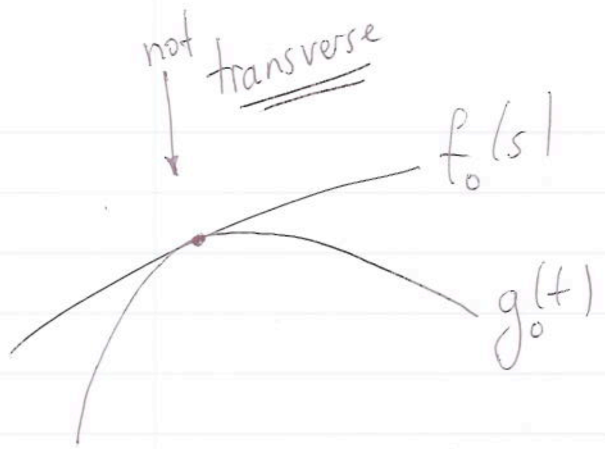
tg vct to curves are <u>not</u> tgt to each other.

IFT

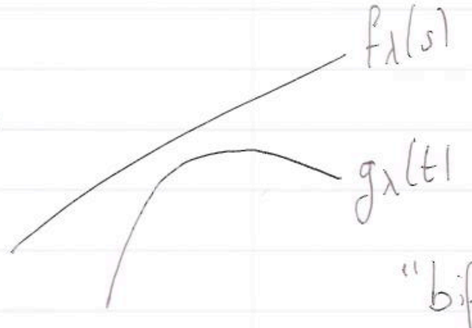
then $\exists!\ s(\lambda), t(\lambda)$ with $s(0) = t(0) = 0$ such that $h_\lambda(s(\lambda), t(\lambda)) = 0$ for λ suff close to 0.

10/2

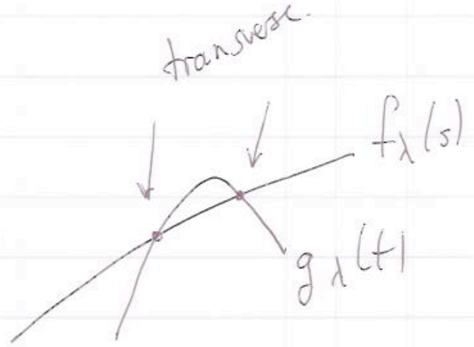
$\lambda = 0$



$\lambda > 0$



$\lambda < 0$



"bifurcation"

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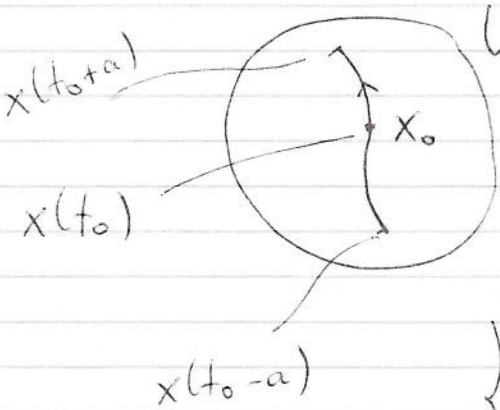
NEW PROBLEM SHEET (nr 4)
IN THE BACK.

Aim: solve $\frac{dx}{dt} = f(x)$ $x \in \mathbb{R}^n$ $x(t_0) = x_0$

in nbh U of x_0



U $|U| = b$

$U \ni x(t)$, $t \in [t_0 - a, t_0 + a]$

$x \in C^0(J, U)$

$$\begin{cases} |f(x) - f(y)| \leq K|x-y| & x, y \in U \\ |f(x)| \leq M & x \in U. \end{cases}$$

---

note $|T(u(t)) - T(v(t))| \leq \sup_{t \in J} |T(u(t)) - T(v(t))|$

$= d(T(u), T(v))$

PLEASE SEE NOTE ON WEBSITE

no!

$\Rightarrow$ $\boxed{d(T(u), T(v)) \leq aK\, d(u,v)}$

metric in $C^0(J, U)$

radius of J

Lipschitz constant of f

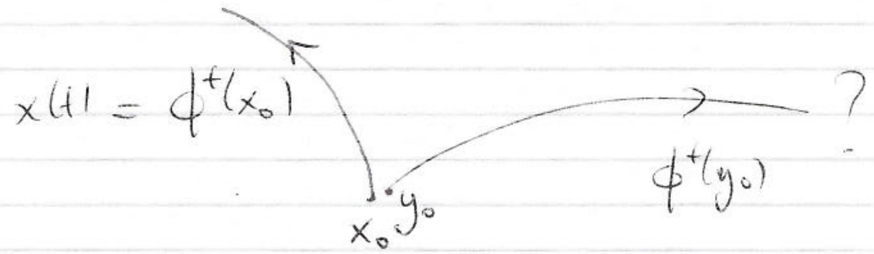
in order for $T: C^0(J,U) \to C^0(J,U)$

to be a contraction $\Rightarrow$ $aK < 1$

12/2

<u>We know:</u> ϕ^t continuous in time

but is $\phi^t: \mathbb{R}^m \to \mathbb{R}^m$ also continuous (wrt $\mathbb{R}^m$) ?



if ϕ^t is continuous then

if $d(y_0, x_0)$ suff small

$\Rightarrow d(x(t), y(t)) < \varepsilon$ for some $|t| < \delta$

small