BSc and MSci EXAMINATIONS (MATHEMATICS)

January 2006

M1S (Test)

Probability and Statistics

- Write your name, College ID, Personal Tutor, and the question number prominently on the front of each answer book.
- Write answers to each question in a separate answer book.
- Credit will be given for all questions attempted, but extra credit will be given for complete or nearly complete answers.
- The question in Section A will be worth $1\frac{1}{2}$ times as many marks as either question in Section B.
- Calculators may not be used.

SECTION A

1. (i) Suppose that events E_1, E_2 and E_3 are exclusive and that event E is a subset of $E_1 \cup E_2 \cup E_3$. Show what form the Law of Total Probability takes for $\mathrm{P}(E)$ in this case.

(ii) Given that $\mathrm{P}(A \mid B)/\mathrm{P}(A) = 2$, find $\mathrm{P}(B \mid A)/\mathrm{P}(B)$.

(iii) The continuous random variable X has density $f(x) = kx^3$ on (0,1), where k is a constant. Find k and $\mathrm{E}(X^2)$, and write down an expression for $F(x)$, the distribution function of X.

(iv) The discrete random variable Y takes values -1, 0 and 1 with respective probabilities $\frac{1}{6}$, $\frac{1}{2}$ and $\frac{1}{3}$. Find an expression for $G(s)$, the probability generating function of Y, and use it to calculate the mean and variance of Y.

(v) The random variable U is selected ('at random') from the uniform distribution on (0,1). Then V is defined as the volume of a rectangular box of sides U, $2U$ and $4U$. Calculate the probability that V is greater than 1.

(vi) The discrete random variables X_1 and X_2 have a joint distribution with means $(\mu_1, \mu_2) = (2, 1)$, variances $(\sigma_1^2, \sigma_2^2) = (4, 1)$ and covariance $\sigma_{12} = -3$. Calculate the mean and variance of $3X_1 - 2X_2$.

SECTION B

2. (a) Box A contains 3 blue beads, 2 green and 2 red. Box B contains 2 blue beads, 3 green and 4 red. One bead is drawn from each box. What is the probability that neither of them is blue? What is the probability that at least one of them is blue? Given that at least one of them is blue, what is the probability that the other is red?

   (b) A carnival game consists of rolling a coin of radius r onto a table marked with a square grid of side a, where $r < a/2$. If the coin makes no contact with any grid line the player wins. Calculate the probability that the player wins. (Hint: what area can the centre of the coin occupy so that the player wins?) If five players compete, what is the probability that at least one will win? If a single player continues to play until he wins, what is the probability that he has to roll five coins?

3. (a) The random variable X takes values 0, 1 and 2 with probabilities $\frac{1}{4}$, $\frac{1}{4}$ and $\frac{1}{2}$. Compute $\mathrm{E}(\frac{1}{X+1})$.

   (b) The annual profit Y of a company is related to the demand X for its product by $Y = a(1 - \mathrm{e}^{-bX})$, and X has an exponential distribution with density $\lambda \mathrm{e}^{-\lambda x}$ on $(0, \infty)$, with $\lambda > 0$. Calculate $\mathrm{P}(Y > c)$. (Note: a, b and c here are positive constants.)

   (c) The table below shows the joint probabilities of the discrete bivariate distribution of X_1 and X_2. Write down the marginal probabilities $p_1(x_1)$ of X_1, and compute the mean and variance of X_1. Calculate the conditional probability distribution of X_1 given that $X_2 = 1$. Are X_1 and X_2 independent?

	$X_1 = 1$	2	3	4
$X_2 = 1$	0.2	0.0	0.2	0.0
2	0.1	0.1	0.1	0.1
3	0.0	0.1	0.0	0.1