PHYSICS 1: MATHEMATICAL ANALYSIS I.

PROBLEMS 8

1. Use the ratio test to determine whether the following two series are convergent

$$\sum_{n=0}^{\infty} \frac{1}{(n+1)!} \qquad \text{and} \qquad \sum_{n=0}^{\infty} \frac{(3-4i)^n}{n!}.$$

2. Show that $y = \tan x$ satisfies the equation

$$\frac{dy}{dx} = 1 + y^2.$$

By repeated differentiation of this result, find the higher derivatives that are required to determine the first three non-zero terms of the Maclaurin series for $\tan x$.

3. Show that there are two stationary values of the function

$$u(x,y) = \frac{x^2 + y^2 + 2x + 1}{x + y}.$$

By considering the second partial derivatives u_{xx}, u_{yy} and u_{xy}, show that one is a maximum and the other is a minimum.

4. Sketch contours (curves of constant u) for the function $u = xy(x+y-1)$ and indicate regions where u is zero, positive and negative respectively. Locate the stationary points of the function and deduce their nature from the contour diagram. Now use the standard method of calculating the sign of $\left(u_{xy}^2 - u_{xx}u_{yy}\right)$ etc at each stationary point to confirm your findings.

STARRED PROBLEM

5* Show that the function

$$u(x,y) = x^4 + 4x^2y^2 - 2x^2 + 2y^2 - 1$$

has three stationary points, two of which are minima, the other being a saddle.