Physics I Mathematical Analysis SOLUTIONS 5.

(1) a) $y' = \sinh x$ $\quad s = \int_0^1 (1+\sinh^2 x)^{1/2}\,dx = \int_0^1 \cosh x\,dx = \sinh 1.$

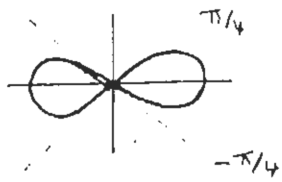
b) $x = \cos t,\ y = \sin t,\ z = t.$ $\quad (ds)^2 = [(-\sin t\,dt)^2 + (\cos t\,dt)^2 + (dt)^2]$

$s = \int ds = \sqrt{2}\int_0^{2\pi} dt = 2\sqrt{2}\,\pi.$

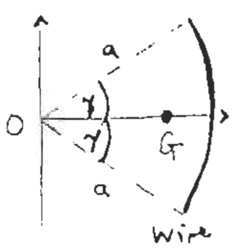
c) $y = x^{3/2}$ $\quad s = \int_0^4 [1 + \frac{9}{4}x]^{1/2}\,dx = \frac{8}{27}(10^{3/2} - 1)$

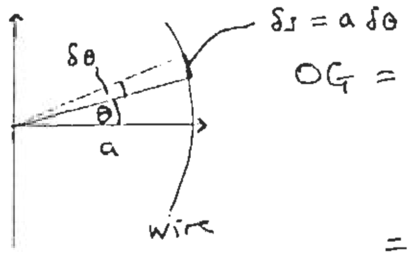
d) $x = \cos^3 t,\ y = \sin^3 t$ $\quad s = 9^{1/2}\int_0^{\pi/2} [\cos^4 t\sin^2 t + \sin^4 t\cos^2 t]^{1/2}\,dt$

$\therefore s = 3\int_0^{\pi/2} \cos t\sin t\,dt = \frac{3}{2}\int_0^{\pi/2} \sin 2t\,dt = 3/2.$

(2) 

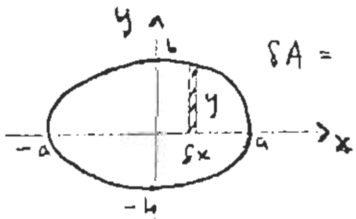
$\text{Area} = \frac{1}{2}\int r^2\,d\theta = \frac{a^2}{2}\int_{-\pi/4}^{\pi/4} \cos 2\theta\,d\theta = \frac{1}{2}a^2$

(3) 



$OG = \dfrac{\int x\rho\,ds}{\int \rho\,ds}$ $\quad \rho$ is mass of wire/unit length.

$= \dfrac{\rho\int_{-\gamma}^{\gamma} a^2\cos\theta\,d\theta}{\rho\int_{-\gamma}^{\gamma} a\,d\theta} = \dfrac{a\sin\gamma}{\gamma}$

(4) 

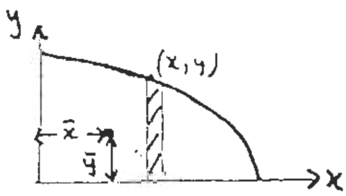
$\delta A = y\,\delta x$

$\text{Area of ellipse} = 4\int_0^a y\,dx$

$= 4b\int_0^a \left(1 - \frac{x^2}{a^2}\right)^{1/2} dx$

Put $x = a\cos\theta$ so $A = -4ab\int_{\pi/2}^{0} \sin^2\theta\,d\theta$ $\qquad \cos 2\theta = 1 - 2\sin^2\theta$

$= \frac{4ab}{2}\int_0^{\pi/2} (1 - \cos 2\theta)\,d\theta = \pi ab.$



Mass of strip

$\delta M = \rho y\,\delta x$

$\therefore \rho A_1\bar{x} = \int x\rho y\,dx$ $\qquad A_1$ area of 1st quadrant. ρ is mass/unit area.

$\therefore A_1\bar{x} = \int_0^a x\,b(1 - x^2/a^2)^{1/2}\,dx$

$\therefore A_1\bar{x} = -a^2 b\int_{\pi/2}^{0} \cos\theta\sin^2\theta\,d\theta = \frac{1}{3}a^2 b$

$A_1 = \frac{\pi ab}{4}$, the area of the 1st quadrant. Hence.

$\bar{x} = \frac{4a}{3\pi}.$ By symmetry $\bar{y} = \frac{4b}{3\pi}.$