M Phys I. Mathematical Analysis

SOLUTIONS TO PROBLEMS 4

① $f(x) = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1} \Rightarrow A(x-1)(x+1) + Bx(x+1) + Cx(x-1) = 2x^2 - x + 2.$

$(A, B, C) = (-2, 3/2, 5/2)$. Hence

$$\int f(x)\,dx = -2\ln|x| + \frac{3}{2}\ln|x-1| + \frac{5}{2}\ln|x+1| + c.$$

② $\sin 3x \cos 5x = \frac{1}{2}[\sin 8x - \sin 2x]$ from trig. formula

$\therefore \quad I = \frac{1}{2}\int (\sin 8x - \sin 2x)\,dx = -\frac{1}{16}\cos 8x + \frac{1}{4}\cos 2x + c$

③ $\overline{\sin x} = \frac{1}{\pi}\int_0^{\pi} \sin x\,dx = -\frac{1}{\pi}\left[\cos x\right]_0^{\pi} = -\frac{1}{\pi}[-1-1] = 2/\pi$

$\overline{\sin^2 x} = \frac{1}{2\pi}\int_0^{2\pi} \sin^2 x = \frac{1}{4\pi}\int_0^{2\pi}(1-\cos 2x)\,dx = \frac{1}{2}$

④ $I_n = \int_0^{\pi/2} \sin^n x\,dx = -\int_0^{\pi/2} \sin^{n-1} x\,d(\cos x)$

$= -\left[\sin^{n-1}x \cos x\right]_0^{\pi/2} + (n-1)\int_0^{\pi/2} \sin^{n-2}x \cos^2 x\,dx \qquad n \geq 2$

$= 0 + (n-1)\int_0^{\pi/2} \sin^{n-2} x\,(1-\sin^2 x)\,dx$

$= (n-1)\left[I_{n-2} - I_n\right]$

Solve for I_n: $\quad n I_n = (n-1) I_{n-2} \qquad n \geq 2$

$I_8 = \frac{7}{8} I_6 = \frac{7}{8}\cdot\frac{5}{6} I_4 = \frac{7}{8}\cdot\frac{5}{6}\cdot\frac{3}{4} I_2 = \frac{7.5.3.1}{8.6.4.2} I_0$

$I_0 = \int_0^{\pi/2} dx = \pi/2$

$\therefore \quad I_8 = \frac{35}{256}\pi.$