PHYSICS 1: MATHEMATICAL ANALYSIS I.
PROBLEMS 4

1. Put into partial fractions and hence find the indefinite integral of

$$f(x) = \frac{2x^2 - x + 2}{x(x-1)(x+1)}.$$

2. By using the trigonometric formula $\sin(A+B) + \sin(A-B) = 2\sin A \cos B$ calculate the indefinite integral

$$I = \int \sin 3x \cos 5x \, dx.$$

3. Recall from the lectures that the mean value $\overline{f}$ of a function $f(x)$ over an interval $0 \le x \le a$ is given by

$$\overline{f} = \frac{\int_0^a f(x)\,dx}{\int_0^a dx}.$$

Find the mean value of $f(x) = \sin x$ in the interval $0 \le x \le \pi$, and of $f(x) = \sin^2 x$ in the interval $0 \le x \le 2\pi$.

4. If

$$I_n = \int_0^{\pi/2} \sin^n x \, dx,$$

where $n \ge 0$ is an integer, show that $I_n = \frac{n-1}{n} I_{n-2}$, for $n \ge 2$. Hence show that

$$I_8 = \int_0^{\pi/2} \sin^8 x \, dx = \frac{35}{256}\pi.$$

STARRED PROBLEMS

5* Calculate the length of the curve

$$y = \frac{x^3}{a^2} + \frac{a^2}{12x},$$

from $x = a/2$ to $x = a$, where a is a positive constant.

6* If

$$I = \int_0^{\pi/2} \frac{\sin^{1/3} x}{\sin^{1/3} x + \cos^{1/3} x}\, dx,$$

use the substitution $x = \pi/2 - y$ to show that

$$I = \int_0^{\pi/2} \frac{\cos^{1/3} x}{\sin^{1/3} x + \cos^{1/3} x}\, dx.$$

Hence show that $I = \pi/4$.