

PHYSICS 1: MATHEMATICAL ANALYSIS I.
PROBLEMS 3

1. Integrate by parts:

(a) $x^3 \sin x$; (b) $\tan^{-1} x$.

2. Evaluate the integrals:

(a) $\int_0^1 (1+x^2)^{-3/2} dx$ (b) $\int_0^\infty (1+e^{2x})^{-1} dx$, (c) $\int_1^{3/2} (2-x)^{-1}(x-1)^{-1/2} dx$.

Hint: In (a) substitute $x = \tan \theta$; in (b) substitute $u = e^{2x}$; in (c) use the substitution $(x-1) = u^2$.

3. Which of the following integrals are convergent?

(a) $\int_0^1 \ln x dx$; (b) $\int_0^1 (x-1)^{-2} dx$;

4. Show that

$$\int x^k \ln x dx = \frac{x^{k+1}}{(k+1)^2} [(k+1) \ln x - 1] + c$$

where c is a constant and $k \neq -1$.

STARRED PROBLEMS

5* Which of the following integrals are convergent?

(a) $\int_1^\infty \ln x dx$; (b) $\int_0^\infty e^{-ax} \sin bx dx$, ($a > 0$).

6* Integrate

(a) $\frac{x^4}{x^2+1}$; (b) $\frac{1}{x \ln x}$.