MPhys I **Mathematical Analysis**

Solutions to Sheet 2

① (i) $\frac{dy}{dx} = 3x^2\cos(5x+1) - 5x^3\sin(5x+1)$ (Product Rule)

(ii) $\frac{dy}{dx} = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x} = \sec x$ (Note $\frac{d}{dx}\sec x = \frac{\sin x}{\cos^2 x} = \sec x \tan x$)

(iii) $\frac{dy}{dx} = \frac{x+1-x}{(x+1)^2} = (x+1)^{-2}$ (Quotient Rule)

② (a) $y^3 = x^3 - xy \to 3y^2\frac{dy}{dx} = 3x^2 - \left(y + x\frac{dy}{dx}\right)$

Hence $\frac{dy}{dx} = \frac{3x^2 - y}{3y^2 + x}$

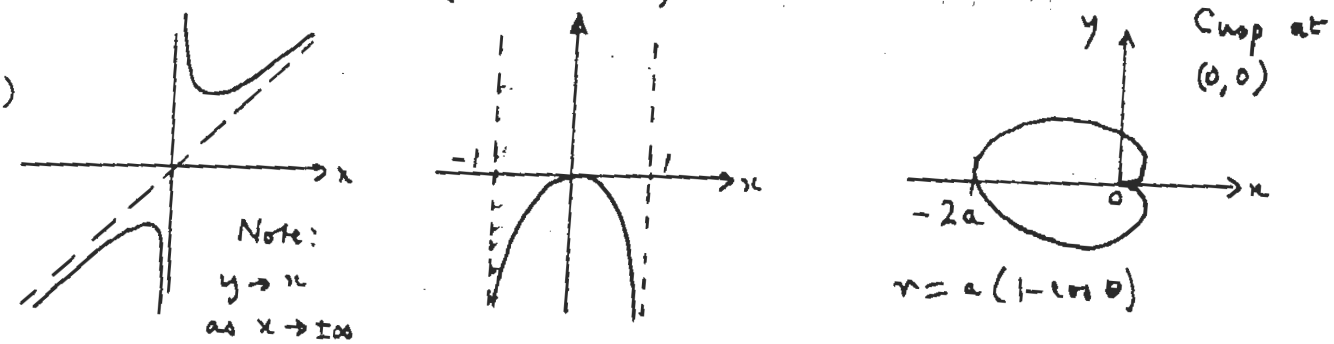
(b) $xe^y = \cos xy$: LHS: $\frac{d}{dx}(xe^y) = e^y + x\frac{dy}{dx}e^y$

RHS: $\frac{d}{dx}(\cos xy) = -\sin(xy)\left(y + x\frac{dy}{dx}\right)$

$\therefore \frac{dy}{dx}\left(xe^y + x\sin xy\right) = -\left(y\sin xy + e^y\right)$

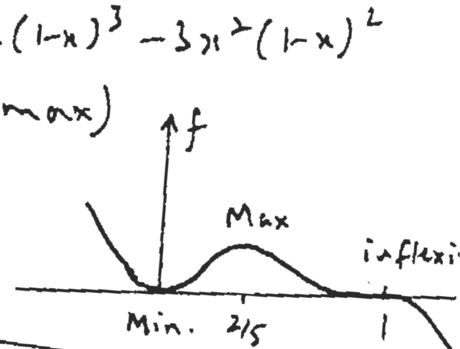
so $\frac{dy}{dx} = -\frac{y\sin xy + e^y}{x(e^y + \sin xy)}$

③ (a)



④ Stationary pts when $f' = 0$: $f'(x) = 2x(1-x)^3 - 3x^2(1-x)^2$

$x = 0$ (min), $x = 1$ (inflexion), $x = 2/5$ (max)



⑤ $r(1+\cos\theta) = 2 \Rightarrow r + x = 2$

because $x = r\cos\theta$. Hence

$r^2 = (2-x)^2$ so $x^2 + y^2 = (2-x)^2$

$\therefore y^2 = 4(1-x)$ (Parabola)