

PHYSICS 1: MATHEMATICAL ANALYSIS I.
PROBLEMS 2

1. Differentiate $x^3 \cos(5x + 1)$; $\ln(\sec x + \tan x)$; $x/(x + 1)$.
2. Find dy/dx when (a) $y^3 = x^3 - xy$; (b) $xe^y = \cos(xy)$.
3. Sketch the graphs of the following:
 - (a) $y = x + 1/x$, ($x \neq 0$);
 - (b) $y = \ln(1 - x^2)$, $-1 < x < 1$;
 - (c) $r = a(1 - \cos \theta)$ where r and θ are polar coordinates and a is a positive constant.

Note: Plane polar co-ordinates (r, θ) are related to Cartesian co-ordinates (x, y) by $x = r \cos \theta$ and $y = r \sin \theta$: hence $r^2 = x^2 + y^2$ and $\theta = \tan^{-1}(\frac{y}{x})$.

4. Find the stationary points of the function $f(x) = x^2(1 - x)^3$ and determine their nature. Sketch the graph $y = f(x)$.
5. If $r(1 + \cos \theta) = 2$, where r and θ are plane polar coordinates, express the equation in terms of cartesian coordinates (x, y) ; show that the graph is a parabola and sketch it.

STARRED PROBLEMS

6* Differentiate $y = \sin^{-1}\{x/(1 + x)\}$ and $y = \sec^{-1}(x)$.

7* Find where the function

$$f(x) = \frac{2x^2 - 5x - 25}{x^2 + x - 2}$$

is discontinuous. Find also the points where it is zero, its limiting values as $x \rightarrow \pm\infty$ and its maximum and minima. Hence sketch its graph.