MATHEMATICAL ANALYSIS SOLUTIONS 1. M. Phys 1.

1) $f(0) = 2$; $f(x^2) = x^4 - 3x^2 + 2$

$f(x+1) = (x+1)^2 - 3(x+1) + 2 = x^2 - x$

$f(x) = 0$ at roots of $x^2 - 3x + 2 = 0$, namely $x = 1$ and 2

so $f(2x) = 0$ at $x = \frac{1}{2}$ and 1.

2) a) $y = 3x + 4 \Rightarrow x = \frac{1}{3}(y-4)$

Hence $f^{-1}(x) = \frac{1}{3}(x-4)$ for all x.

b) $y = 2x + x^2 \quad (0 < x < 1)$ so the range is $0 < y < 3$.

Solve the quadratic in x i.e. $x^2 + 2x - y = 0$,

which gives $x = -1 \pm (1+y)^{1/2}$. Note that the

2nd root $x = -1 - (1+y)^{1/2}$ is negative and out

of the domain $0 < x < 1$. We reject this root,

leaving $f^{-1}(x) = -1 + (1+x)^{1/2}$ for $0 < x < 3$

3) (a) Neither (b) Even (c) Neither (d) Odd

4) (a) $\lim\limits_{x \to \infty} \dfrac{x^2+1}{x^2-1} = \lim\limits_{x \to \infty} \dfrac{1 + 1/x^2}{1 - 1/x^2} = 1$

(b) $\lim\limits_{x \to 0} \dfrac{(1+x)^{1/2} - (1-x)^{1/2}}{x} = \lim\limits_{x \to 0} \frac{1}{x}\left[(1 + \tfrac{1}{2}x + \dots) - (1 - \tfrac{1}{2}x \dots)\right]$ Binomial Thm.

$= 1$

5) (a) $\lim\limits_{x \to \infty} x \sin\left(\frac{1}{x}\right) = \lim\limits_{y \to 0} \left(\dfrac{\sin y}{y}\right) = 1 \qquad (y = 1/x)$

(b) $\lim\limits_{x \to 1} \dfrac{x^9 + x - 2}{x^4 + x - 2} = \lim\limits_{h \to 0} \dfrac{(1+h)^9 + (1+h) - 2}{(1+h)^4 + (1+h) - 2}$

$= \lim\limits_{h \to 0} \dfrac{(1 + 9h + \dots) + (1+h) - 2}{(1 + 4h + \dots) + (1+h) - 2} = \dfrac{10}{5} = 2$

Alternatively, L'H Rule gives,

$\lim\limits_{x \to 1} \dfrac{x^9 + x - 2}{x^4 + x - 2} = \lim\limits_{x \to 1} \dfrac{9x^8 + 1}{4x^3 + 1} = \dfrac{10}{5} = 2.$