PHYSICS 1: MATHEMATICAL ANALYSIS I.
PROBLEMS 1

1. If $f(x) = x^2 - 3x + 2$, find $f(0), f(x^2), f(x+1)$. For what values of x does $f(x) = 0$? For what values of x does $f(2x) = 0$?

2. Find the inverse of each of the functions:

$$\text{(a)} \quad f(x) = 3x + 4, \quad \text{all real } x;$$

$$\text{(b)} \quad f(x) = 2x + x^2, \quad 0 < x < 1.$$

3. Are the following functions even, odd or neither?

$$\text{(a)} \quad x^2 + 2\sin x; \qquad \text{(b)} \quad (1+x^4)^{-1}\cos 3x;$$

$$\text{(c)} \quad x + |x|; \qquad \text{(d)} \quad \sin^3 x.$$

4. Evaluate the following limits:

$$\text{(a)} \quad \lim_{x \to \infty} \frac{x^2+1}{x^2-1}; \qquad \text{(b)} \quad \lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}.$$

5. Evaluate the limits:

$$\text{(a)} \quad \lim_{x \to \infty} x\sin\left(\frac{1}{x}\right); \qquad \text{(b)} \quad \lim_{x \to 1} \frac{x^9 + x - 2}{x^4 + x - 2}.$$

Hint for (b): Either use L'Hôpital's Rule or put $x = 1 + h$ and use the binomial expansion.

Starred Question

6* Given the definitions (from the lectures) of the hyperbolic functions

$$\cosh x = \frac{(e^x + e^{-x})}{2} \qquad \sinh x = \frac{(e^x - e^{-x})}{2} \qquad \tanh x = \frac{\sinh x}{\cosh x}$$

show that

1. $\cosh^2 x - \sinh^2 x = 1$,
2. $\cosh^2 x + \sinh^2 x = \cosh 2x$,
3. $\sinh(x_1 + x_2) = \sinh x_1 \cosh x_2 + \sinh x_2 \cosh x_1$,
4. $\frac{d}{dx}\tanh x = \operatorname{sech}^2 x \qquad (\operatorname{sech} x = \frac{1}{\cosh x})$.

Note the differences in the signs in 1) and 2) from the trigonometric cases.