M. PHYS. 1: MATHEMATICAL ANALYSIS

LECTURER: Prof. A. Parry (20 lectures)

AIMS: To ensure that all students acquire the mathematical knowledge and skills required for their First year Physics course and to prepare them for future years.

OBJECTIVES: These are determined in consultation with the Department of Physics. They are specified in the course syllabus (laid out below), the associated problem sheets and the course examinations. The principal topics of this First Year course are:

1. Functions and limits

2. Differentiation and curve sketching

3. Integration and its applications

4. Partial differentiation

5. Power series; max/min/saddle points of functions of two variables

SYLLABUS

- **FUNCTIONS & LIMITS:** Simple definition; polynomials and trigonometric functions; odd and even functions; inverse functions; continuous functions; Heaviside function. Logarithmic, exponential and hyperbolic functions. Limits, special case $x^{-1}\sin x$ as $x \to \infty$ and others. Use of l'Hôpital's rule.

- **DIFFERENTIATION & CURVE SKETCHING:** Definition as limit; derivative of $x^n, \sin x, \cos x, \ln(x), \exp x$; derivative of sum, products and quotients; implicit differentiation when $F(x, y) = 0$; derivative of inverse function, i.e. $dy/dx = 1/(dx/dy)$; function of a function, i.e. $dy/dt = (dy/dx)(dx/dt)$. Stationary points, maxima, minima and inflexion points. Symmetries; features near important points, x small and x large, asymptotes. Curves given in polar coordinates r and θ; polar form of ellipse, parabola, hyperbola.

- **INTEGRATION & APPLICATIONS:** Definition as limit, using area below curve. Fundamental theorem of calculus, i.e. $dF/dx = f(x)$, where $f(x) = \int_{x_0}^{x} f(t)dt$, (an indefinite integral of f(x)). Techniques for integration: use of indefinite integral, substitution, integration by parts. Mean value. Improper and infinite integrals. Applications of integration: arc length, area of plane surface in cartesian and polar forms, volume and surface area of bodies of revolution, mass centre, moments of inertia.

- **PARTIAL DIFFERENTIATION:** Function of two variables: $u = f(x, y)$; maxima, minima, saddle points through sketch contour lines (level surfaces) $u = $ constant. Definition of $\partial u/\partial x = (\partial u/\partial x)_y = u_x$, (like ordinary derivative with y temporarily held constant) and $\partial u/\partial y = u_y$. Differential relations $\delta u \approx (\partial u/\partial x)\delta x + (\partial u/\partial y)\delta y$, $du = (\partial u/\partial x)dx + (\partial u/\partial y)dy$. Generalisation to any number of independent variables. Higher order partial derivatives; equality (usually) of mixed derivatives, i.e. $\partial^2 u/\partial x\partial y = \partial^2 u/\partial y\partial x$. Function of a function, chain rule: $\partial u/\partial s = (\partial u/\partial x)(\partial x/\partial s) + (\partial u/\partial y)(\partial y/\partial s)$ etc. Condition for $Fdx + Gdy$ to be exact differential.

- **POWER SERIES:** Meaning of convergence, alternating series test, ratio test: term-by-term integration and differentiation. Taylor series, Maclaurin series. Taylor series in two variables. Double Taylor series; maxima, minima and saddle points of functions of two variables.

- *Course delivery*: A combination of blackboard, OHP and printed notes.

- *Class Arrangements*: Classwork once per week after the first lecture, with Tutors from the Mathematics Department and the Physics Department. 8 problem sheets.

- *Assessment:* Examination: One 3 hour written examination at the end of the session with contributed questions from the other Mathematics course run by the Physics Department.

RECOMMENDED TEXTS

1.	M. L. BOAS	Math. Methods in the Physical Sciences	Wiley.
2.	G. STEPHENSON	Math. Methods for Science Students	Longmans.
3.	A. JEFFERY	Maths for Engineers & Scientists	Nelson.
4.	E. KREYSZIG	Advanced engineering Maths	Wiley.

Notes: 1 & 2 are clear and easy to follow; 1 particularly useful for physicists; 4 is more advanced and comprehensive.

h:teach/mphys/mpsyll