Imperial College London

BSc and MSci EXAMINATIONS (MATHEMATICS)

January 2007

M1GLA (Test)

Geometry and Linear Algebra

- Write your name, College ID, Personal Tutor, and the question number prominently on the front of each answer book.
- Write answers to each question in a separate answer book.
- Credit will be given for all questions attempted, but extra credit will be given for complete or nearly complete answers.
- The question in Section A will be worth $1\frac{1}{2}$ times as many marks as either question in Section B.
- Calculators may not be used.

SECTION A

1. (a) Find the perpendicular distance from the point $(2, 3)$ to the line

$$L = \{(-1, -1) + \lambda(4, 5) | \lambda \in \mathbb{R}\}.$$

(b) Find the point of intersection of the line L from (a) with the line through the point $(2, 3)$ which is perpendicular to L.

(c) What is the focus of the parabola $y = x^2$?

(d) What is the directrix of the parabola $y = x^2$?

(e) What type of conic is represented by the equation $x_1^2 - 4x_1x_2 + 4x_2^2 - 9 = 0$?

(f) Find the value of a such that the following system has infinitely many solutions:

$$\begin{aligned} x_1 + x_2 - x_3 &= 1 \\ 2x_1 + ax_3 &= 2 \\ 2x_2 - 2x_3 &= a \end{aligned}$$

(g) Find the inverse of the matrix

$$\begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

(h) Let $A = \begin{pmatrix} -1 & 0 \\ 3 & 2 \end{pmatrix}$. Find an invertible 2×2-matrix P such that $P^{-1}AP$ is diagonal.

(i) Find A^{2007}, where A is as in (h).

(j) Let $A = (0, 2, 1)$, $B = (1, 3, 1)$, $C = (1, 4, 0)$ be three points in $\mathbb{R}^3$. Find a unit normal vector to the plane through A, B and C.

SECTION B

2. (a) Prove that the number of solutions of any system of linear equations is 0, 1, or infinity.

   (b) For any real number a reduce the conic $x_1^2 + 6\sqrt{3}x_1x_2 + 7x_2^2 = a$ to the standard form, and hence determine its type. You can use any method you like.

3. (a) Let A be a symmetric 3×3-matrix, and let $v_1,\ v_2 \in \mathbb{R}^3$ be column vectors such that $Av_1 = \lambda_1 v_1,\ Av_2 = \lambda_2 v_2,\ \lambda_1 \neq \lambda_2$. Prove that $v_1.v_2 = 0$.

   (b) Give a definition of an orthogonal matrix.

   (c) Find real numbers a, b, c such that the following matrix is an orthogonal matrix with determinant 1:

$$\begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 \\ -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ a & b & c \end{pmatrix}.$$