Imperial College London

UNIVERSITY OF LONDON

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BSc and MSci EXAMINATIONS (MATHEMATICS)

May-June 2008

M1GLA

Geometry and Linear Algebra

Setter's signature	Checker's signature	Editor's signature
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BSc and MSci EXAMINATIONS (MATHEMATICS)

May-June 2008

This paper is also taken for the relevant examination for the Associateship.

M1GLA

Geometry and Linear Algebra

Date: examdate Time: examtime

Credit will be given for all questions attempted but extra credit will be given for complete or nearly complete answers.

Calculators may not be used.

1. In this question you are *not* asked to prove anything. Writing your answer is enough.

   (a) Briefly explain how, given the unit length, to construct $\frac{1}{3}\sqrt{3}$ using only ruler and compass.

   (b) Let L be the line perpendicular to the line $ax + by = c$ and passing through the point (a, b). Find the perpendicular distance of $(0, 0)$ from L.

   (c) What is the focus of the parabola $x^2 = -4y$?

   (d) Give an example of an orthogonal 2×2 matrix without zero entries.

   (e) Find a unit normal vector to the plane through $A = (1, 2, 3)$, $B = (2, 3, 1)$, $C = (3, 1, 2)$ in $\mathbb{R}^3$.

2. (a) Use Gaussian elimination to solve the system of linear equations for arbitrary a, b, $c \in \mathbb{R}$:

$$\begin{aligned} x_1 + x_2 + x_3 &= a \\ x_1 + 2x_2 + 4x_3 &= b \\ x_1 + 3x_2 + 7x_3 &= c \end{aligned}$$

   (b) Find the inverse of the matrix

$$\begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}$$

   for arbitrary a, b, $c \in \mathbb{R}$, using any method you like.

   (c) Using any method you like find

$$\begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}^{19}$$

3. (a) Find the eigenvalues and the eigenvectors of the matrix

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{pmatrix}$$

   (b) Find a 2×2 matrix with *real* entries whose eigenvalues are $1 + i$ and $1 - i$.

   (c) Determine the type of the conic $x_1^2 + 2\lambda x_1 x_2 - x_2^2 = \lambda$ for all values of $\lambda \in \mathbb{R}$. You can use any results from the lectures without proof provided you state them explicitly.

4. In this question you can use any results from the lectures without proof provided you state them explicitly.

   (a) Prove that two planes in $\mathbb{R}^3$ with a common point have a common line.

   (b) Prove that the vector product of (x_1, x_2, x_3) and (y_1, y_2, y_3) is perpendicular to both these vectors.

   (c) Find a vector perpendicular to the lines L_1 and L_2 in $\mathbb{R}^3$, where L_1 contains $A = (1, 2, 3)$ and $B = (4, -1, 2)$, and L_2 contains $C = (-1, 2, 1)$ and $D = (1, 1, 1)$.

   (d) Find the area of the triangle ABC, where A, B, C are as in part (c) of this question.