UNIVERSITY OF LONDON

Course:	M1GLA
Setter:	Skorobogatov
Checker:	Liebeck
Editor:	Ivanov
External:	Cremona
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BSc and MSci EXAMINATIONS (MATHEMATICS)

May-June 2007

M1GLA

Geometry and Linear Algebra

Setter's signature	Checker's signature	Editor's signature
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BSc and MSci EXAMINATIONS (MATHEMATICS)

May-June 2007

This paper is also taken for the relevant examination for the Associateship.

M1GLA

Geometry and Linear Algebra

Date: examdate Time: examtime

Credit will be given for all questions attempted but extra credit will be given for complete or nearly complete answers.

Calculators may not be used.

1. (a) Given three points on the circumference of a circle, explain how to construct the centre of the circle using only ruler and compass. (3 or 4 sentences will suffice.)

   (b) Let $a = (a_1, a_2)$, $b = (b_1, b_2) \in \mathbb{R}^2$. Find the equation of the perpendicular bisector of the line ab. (Justify your answer.)

   (c) Let (a_1, a_2), (a_3, a_4), $(a_5, a_6) \in \mathbb{R}^2$ be non-collinear points whose coordinates are rational numbers, that is, for each $i = 1, \ldots, 6$ we can write $a_i = \frac{m}{n}$, where m and n are integers, and $n \neq 0$. Prove that the centre of the circle passing through (a_1, a_2), (a_3, a_4), (a_5, a_6) is a point with rational coordinates. (You are not asked to find these coordinates explicitly.)

   (d) Find the equation of the circle passing though the points $(1, 1)$, $(3, -1)$, $(0, \sqrt{3} - 1)$.

2. (a) Define the determinant of a 3×3-matrix.

   (b) Find the inverse of the matrix $\begin{pmatrix} \frac{1}{3} & -\frac{2}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{pmatrix}$ using any method you like.

   (c) For all $\lambda \in \mathbb{R}$ solve the system of equations

$$\begin{aligned} \lambda x_1 + x_2 + x_3 &= 1 \\ x_1 + \lambda x_2 + x_3 &= 1 \\ x_1 + x_2 + \lambda x_3 &= 1 \end{aligned}$$

3. (a) Reduce the conic $-11x_1^2 + 24x_1x_2 - 4x_2^2 = 20$ to the standard form by rotation, and hence determine its type. (You can use any method you like.)

   (b) Give a definition of an ellipse in terms of its focus, directrix and eccentricity.

   (c) Find the two foci of the ellipse $2x_1^2 + 3x_2^2 = 5$. (Proof is not required, just quote the results that you use.)

4. (a) Give a definition of an eigenvalue and an eigenvector.

   (b) Prove that any symmetric 2×2-matrix has two orthogonal eigenvectors.

   (c) Find all 3×3-matrices A such that $AB = BA$, where $B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

5. (a) Define the vector product $a \times b$ of two vectors $a,\ b \in \mathbb{R}^3$.

   (b) State and prove the theorem expressing the area of the parallelogram whose sides are vectors $a,\ b \in \mathbb{R}^3$ in terms of $a \times b$.

   (c) Find an equation of the plane passing through the points $(1, 1, 1)$, $(2, 3, -1)$, $(3, -1, -1)$. (You can use any method you like.)

   (d) Find the perpendicular distance from the point $(0, 0, 0)$ to the plane in part (c).