## UNIVERSITY OF LONDON

## B.Sc., B.Eng. and M.Eng. Examination 2000

## Parts III and IV

For internal students of Imperial College of Science, Technology and Medicine.
This paper also forms part of the examination for the Associateship.

## PROJECT MANAGEMENT

For Chemical Engineering, Information Systems Engineering and Mechanical Engineering Students.

Wednesday $10^{\text {th }}$ May 2000, 14:30-16:00
Closed Book
ANSWER QUESTION 1 (40\%) AND
ANY TWO OTHER QUESTIONS (30\% EACH)
If more than three questions are answered only question one and the first two other questions answered will be marked

Marks will be deducted in this examination if there is insufficient written explanation

## Question 1

A project is specified by the following activities:

| Activity | Immediate <br> predecessor(s) | Duration <br> (weeks) |
| :---: | :---: | :---: |
| A | - | 4 |
| B | - | 5 |
| C | A, B | 9 |
| D | C,A | 7 |
| E | - | 3 |
| F | C, D, E | 5 |
| G | E,F | 4 |
| H | G | 7 |
| I | C | 8 |

(a) Draw an activity on node network to represent the project.
(b) Calculate the earliest start, latest start, earliest finish and latest finish times for each activity.
(c) What is the minimum project completion time; the critical activities and the critical path(s)?
(d) What is meant by the terms total float, free float and independent float. Calculate the total float, free float and independent float for each activity. Comment on the differences (if any) between these float figures.
(e) What effect will each of the following changes (when considered separately) have on the completion time of the overall project? Give reasons for your answer.
(i) Activity I is delayed by 3 weeks
(ii) Activity C is completed in 8 weeks.
(f) Clearly show how the project network you have drawn in (a) above needs to be amended to represent the following conditions:
(1) There must be a time lag of at least 3 weeks between the end of I and the end of G.
(2) There must be a time lag of at least 11 weeks between the start of A and the start of $D$.

Explain why the amendments you have given correctly represent the above conditions. In neither of these two cases do you need to recalculate the overall project completion time.

## Question 2

Consider the following project:

| Activity | Immediate <br> predecessor(s) | Optimistic <br> time (days) | Most likely <br> time (days) | Pessimistic <br> time (days) |
| :---: | :---: | :---: | :---: | :---: |
| A | - | 2 | 4 | 8 |
| B | - | 3 | 7 | 9 |
| C | B,E | 6 | 9 | 10 |
| D | F | 4 | 5 | 7 |
| E | D | 3 | 3 | 3 |
| F | - | 5 | 5 | 5 |
| G | A,C | 6 | 8 | 12 |

(a) Draw an activity on arc network to represent this project.
(b) Using this activity on arc network what is the expected project completion time, the critical activities and the critical path(s)?
(c) Calculate the total float for each non-critical activity.
(d) By making use of the $\mathrm{N}(0,1)$ tables provided calculate the probability that:
(1) the project will take more than 32 days
(2) the project will be completed within 29 days.

## Question 3

The activities in a small project are as below:

| Activity | Immediate <br> predecessor(s) | Normal <br> Time (days) | Crash <br> Time <br> (days) | Normal <br> Cost <br> $\left(£{ }^{\prime} 000\right)$ | Crash <br> Cost <br> $\left(£{ }^{\prime} 000\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A | - | 2 | 1 | 4 | 5 |
| B | - | 7 | 5 | 3 | 8 |
| C | A,B | 3 | 2 | 5 | 7 |
| D | C | 7 | 7 | 2 | 2 |
| E | D | 8 | 4 | 5 | 13 |
| F | E | 9 | 5 | 9 | 21 |
| G | D | 2 | 2 | 8 | 8 |

(a) Write down a linear programming formulation of the above project which represents the problem of completing the project in precisely T days at minimum total cost. Explain clearly why your formulation correctly represents this problem.
(b) Suppose now that every day over a target completion time $\mathrm{T}^{*}$ means a penalty cost of $£ 7000$ is incurred, but every day the project completes under $\mathrm{T}^{*}$ means a bonus of $£ 9000$ is incurred. Clearly indicate how, using linear programming, the problem of determining the optimal project completion time subject to these penalties and rewards, can now be solved by considering just TWO linear programs.
(c) If you did not know about linear programming then how might you crash this project. Illustrate your approach by crashing the project by one day. What complications might arise if you continued to use your approach to crash extra days?

## Question 4

Suppose that we are considering seven possible projects, each of which runs for a number of years. Each project has different capital requirements over its lifetime, as shown in the table below.
$\left.\begin{array}{|l|l|l|l|l|l|l|l|}\hline \text { Project } & \text { Profit } \\ & (£ \mathrm{fm}) & \text { Lifetime } \\ \text { (years) }\end{array}\right)$

For example, project 4 has a lifetime of 3 years, requiring 0.8 of capital in year $3,0.7$ of capital in year 4 and 0.9 of capital in year 5 (its final year). This project has a profit of 0.4 at the end of its life. The available capital in each year is also shown above. For example there is 1.5 of capital available in year 3 .

Formulate the problem of deciding which projects to choose as a linear integer program involving zero-one (binary) variables. Explain clearly why your formulation correctly represents the problem.

Clearly indicate how you would amend your formulation to represent the following conditions:
(1) $25 \%$ of the unused capital in any year can be carried forward as capital available in the next year
(2) projects 2 and 4 are mutually exclusive
(3) project 3 could be started either in year 1, (as above), or in year 2 or in year 3. In each case the capital requirements in each year, and the profit achieved, would be as given above. For example if project 3 started in year 2 then it would require 0.6 of capital in year 2 and 0.9 of capital in year 3 and would give a profit of 0.7 when it finishes.

## $\mathbf{N}(0.1)$ values

|  | 0. | 0.0 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.500 | 0.504 | 0.508 | 0.512 | 0.516 | 0.520 | 0.524 | 0.528 | 0.532 | 0.536 |
|  | 0.540 | 0.54 | 0.548 | 0.552 | 0.556 | 0.560 | 0.564 | 0.567 | 0.571 | 0.5 |
|  | 0.579 | 0.583 | 0.587 | 0.591 | 0.595 | 0.599 | 0.603 | 0.606 | 0.610 |  |
|  | 0.618 | 0.622 | 0.626 | 0.62 | 0.633 | 0.63 | 0.64 | 0.644 | 0.648 |  |
|  | 0.655 | 0.65 | 0.663 | 0.666 | 0.670 | 0.67 | 0.6 | 0.681 | 0.68 |  |
|  | 0.691 | 0.695 | 0.698 | 0.702 | 0.705 | 0.70 | 0.71 | 0.716 | 0.7 |  |
|  | 0.726 | 0.72 | 0.732 | 0.73 | 0.73 | 0.74 | 0.74 | 0.749 | 0.752 |  |
|  | 0.758 | 0.76 | 0.76 | 0.76 | 0.77 | 0.7 | 0.77 | 0.779 | 0.782 |  |
|  | 0.788 | 0.791 | 0.794 | 0.797 | 0.800 | 0.802 | 0.805 | 0.808 | 0.811 | . 8 |
|  | 0.8 | 0.8 | 0. | 0. | 0.826 | 0. | 0. | 0.834 | 0.8 |  |
|  | 0.8 | 0.8 | 0.8 | 0.84 | 0.8 | 0.8 | 0.8 | 0.858 | 0.860 |  |
|  | 0.864 | 0.86 | 0.869 | 0.8 | 0.873 | 0.8 | 0.87 | 0.879 | 0.881 |  |
| 1.2 | 0.8 | 0.8 | 0.889 | 0. | 0. | 0. | 0. | 0.898 | 0. | 0.901 |
| 1.3 | 0.90 | 0.90 | 0.90 | 0.90 | 0.9 | 0.9 | 0.9 | 0.915 | 0.916 |  |
|  | 0.919 | 0. | 0.922 | 0. | 0. | 0. | 0.9 | 0.929 | 0.9 | 0.932 |
| 1.5 | 0.933 | 0.934 | 0.936 | 0.937 | 0.938 | 0. | 0. | 0.942 | 0.943 |  |
| 1.6 | 0.94 | 0.9 | 0.947 | 0. | 0.949 | 0. | 0. | 0.953 | 0.9 |  |
|  | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0. | 0.962 | 0.962 |  |
| 1.8 | 0.964 | 0. | 0.966 | 0. | 0.967 | 0. | 0. | 0 | 0. | 0.971 |
| 1.9 | 0.97 | 0.9 | 0.973 | 0.9 | 0.974 | 0.9 | 0.9 | 0.976 | 0.9 |  |
| 2.0 | 0.97 | 0.97 | 0.978 | 0.97 | 0.979 | 0.98 | 0.98 | 0.981 | 0.981 |  |
| 2.1 | 0.9 | 0. | 0.983 | 0. | 0. | 0. | 0. | 0.985 | 0. |  |
| 2.2 | 0.986 | 0.98 | 0.987 | 0. | 0. | 0. | 0. | 0.988 | 0.9 |  |
| 2.3 | 0.989 | 0.990 | 0.990 | 0.990 | 0.990 | 0.9 | 0.99 | 0.991 | 0.991 |  |
|  | 0. | 0. | 0.992 | 0. | 0. | 0. | 0.9 | 0.993 | 0.993 |  |
| 2. | 0.994 | 0.9 | 0.994 | 0.9 | 0.994 | 0. | 0.99 | 0.995 | 0.995 |  |
| 2.6 | 0.995 | 0.995 | 0.996 | 0.996 | 0.996 | 0.99 | 0.99 | 0.996 | 0.996 |  |
| 2. | 0.997 | 0.997 | 0.997 | 0.99 | 0.99 | 0.99 | 0.99 | 0.997 | 0.997 |  |
| 2.8 | 0.997 | 0.998 | 0.998 | 0.9 | 0.998 | 0.9 | 0.9 | 0.998 | 0.998 |  |
| 2.9 | 0.998 | 0.998 | 0.998 | 0.998 | 0.998 | 0. | 0.99 | 0.999 | 0.999 |  |
| 3.0 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.99 | 0.999 | 0.999 | 0.999 |  |
| 3.1 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.99 | 0.99 | 0.999 | 0.999 |  |
| 3.2 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 |  |
| 3.3 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 3.4 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |  |
| 3.5 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.0 |

